

U46 Curriculum
U-46 Third Grade Mathematics

Module 1 – Properties of Multiplication and Division and Solving Problems with Units of 2-5 and 10

Domain(s): Operations and Algebraic Thinking

Trimester(s): 1

Transfer: *Students will apply...*

1. Knowledge of multiplication and division to solve real-world problems.
2. Problem-solving skills to find the unknown in both multiplication and division problems.
3. Properties of operations to identify arithmetic patterns.
4. The use of equal-sized groups and arrays models to multiplication and division situations.

Equal groups Unknown products problem situation example: There are 5 bags with 3 apples in each bag for the field trip. How many apples in all?
 $5 \times 3 = ?$

Understandings: *Students will understand that...*

1. Visual images and numerical patterns of multiplication and division will assist in solving problems.
2. Modeling multiplication and division problems based upon their problem-solving structure can help in finding solutions.
3. The Properties of Operations will help in performing computation as well as in problem-solving situations. (Distributive Property of Multiplication, Commutative Property of Multiplication, Identity, and Zero.)

Essential Question(s):

1. How do modeling multiplication and division problems help in finding solutions?
2. How can the strategy of breaking apart (decomposing) numbers make multiplication easier to understand?
3. How can we use multiplication to solve division problems?
4. How do multiples and factors relate to multiplication and division?
5. What are the Properties of Operations?

Knowledge: *Students will know...*

1. Multiplication and division facts.
2. Problem-solving structures for arrays and for equal groups

Skill: Students will be able to do...

- Interpret products of whole numbers as the total number of objects in “so many” groups of “so many” objects each. (3.OA.1)
- Interpret whole-number quotients of whole numbers as the number of objects in *each* share or as a number of *equal* shares. (3.OA.2)
- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3)
- Determine the unknown whole number in a multiplication and division equation relating three whole numbers. (3.OA.4)
- Apply properties of operations as strategies to multiply and divide. (3.OA.5)
- Understand division as an unknown-factor problem. (3.OA.6)
- Fluently multiply and divide within 100, using various strategies. (3.OA.7)
- Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (3.OA.8)
- Identify arithmetic patterns, and explain them using properties of operations. (3.OA.9)

Clusters/Standards:

Focus Grade Level Standards

Represent and solve problems involving multiplication and division.¹

- 3.OA.1** Interpret products of whole numbers, e.g., interpret 5×7 as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as 5×7 .*
- 3.OA.2** Interpret whole-number quotients of whole numbers, e.g., interpret $56 \div 8$ as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$.*
- 3.OA.3** Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4** Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*

Understand properties of multiplication and the relationship between multiplication and division.²

¹ Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

² Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.) *Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)³*

3.OA.6 Understand division as an unknown-factor problem. *For example, find $32 \div 8$ by finding the number that makes 32 when multiplied by 8.*

Multiply and divide within 100.⁴

3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.⁵

3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)

Focus Standards for Mathematical Practice

MP.1 **Make sense of problems and persevere in solving them.** Students model multiplication and division using the array model. They solve two-step mixed word problems and assess the reasonableness of their solutions.

MP.2 **Reason abstractly and quantitatively.** Students make sense of quantities and their relationships as they explore the properties of multiplication and division and the relationship between them. Students decontextualize when representing equal group situations as multiplication, and when they represent division as partitioning objects into equal shares or as unknown factor problems. Students contextualize when they consider the value of units and understand the meaning of the quantities as they compute.

MP.3 **Construct viable arguments and critique the reasoning of others.** Students represent and solve multiplication and division problems using arrays and equations. As they compare methods, they construct arguments and critique the reasoning of others. This practice is particularly exemplified in daily application problems and problem-solving specific lessons in which students solve and reason with others about their work.

³ The Associative property is addressed in Module 3.

⁴ Limited to factors of 2–5 and 10 and the corresponding dividends in this module.

⁵ In this module, problem solving is limited to multiplication and division, and limited to factors of 2–5 and 10 and the corresponding dividends. 3.OA.9 is addressed more specifically in Module 3.

MP.4 Model with mathematics. Students represent equal groups using arrays and equations to multiply, divide, add, and subtract.

MP.7 Look for and make use of structure. Students notice structure when they represent quantities by using drawings and equations to represent the commutative and distributive properties. The relationship between multiplication and division also highlights structure for students as they determine the unknown whole number in a multiplication or division statement.

WIDA Standards

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Concrete models of multiplication and division processes.
- Anchor Charts highlighting mathematical vocabulary specific to module.
- Repeated verbalization of processes along with an Anchor Chart highlighting terms and steps.

Student Learning Experiences / Tasks:

Engage NY Module 1

- Warm Up Activities
- Sprints
- Problem Sheets
- Number Bond Dash
- Lesson Activities
- RDW or Read, Draw, Write
- Problem Sets
- Exit Tickets
- Homework

Academic Vocabulary

Critical Terms:

multiplication
decomposing
array
row
column
multiple
product
factor
divide
division
divisor
dividend
quotient
remainder
equal groups
number of groups
size of a group
equal shares
fact family/related facts
distributive property
commutative property

Supplemental Terms:

inverse operation
zero property
identity
equation
parentheses
rotate

Instructional Resources:

Engage NY <http://www.engageny.org/resource/grade-3-mathematics-module-1>

Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

<http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm>

U46 Curriculum
U-46 Third Grade Mathematics

Module 2 – Place Value and Problem Solving with Units of Measure
Domain(s): Number and Operations in Base Ten/Measurement and Data
Trimester(s): 1

Transfer: *Students will apply...*

1. Knowledge of time to real-world problem solving situations.
2. Knowledge and skills to perform real world tasks such as estimating distance/mileage; calculating grocery bills by rounding prices and adding the estimates; etc.

Understandings: *Students will understand that...*

1. Elapsed time is the interval of time, given a specific unit, from a starting time to an ending time.
2. Metric measurement units are related to place value concepts/multiples of 10.
3. Rounding is a method of approximating an answer.

Essential Question(s):

1. How can understanding the relationship between addition and subtraction aid us in problem solving?
2. How does elapsed time help us to plan and organize real life responsibilities?
3. How does metric measurement connect to multiples of 10?
4. How is rounding an efficient method of estimating?
5. Why and when would we round?

Knowledge: *Students will know...*

1. Addition and subtraction computation and problem solving strategies.
2. A.M. represents time from midnight to noon.
3. P.M. represents time from noon to midnight.
4. 60 min = 1 hour.
5. Metric measurements units for liquid volume and weight.
6. When to round in a real-life situation.

Skill: Students will be able to do...

1. Tell and write time to the nearest minute. (3.MD.1)
2. Solve word problems involving elapsed time. (3.MD.1)
3. Use a number line or clocks to model elapsed time and record calculations. (3.MD.1)
4. Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (3.MD.2)
5. Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (3.MD.2)
6. Use place value understanding to round whole numbers to the nearest 10 or 100. (3. NBT.1)
7. Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction. (3. NBT.2)

Clusters/Standards:

Focus Grade Level Standards

Use place value understanding and properties of operations to perform multi-digit arithmetic.¹

- 3.NBT.1** Use place value understanding to round whole numbers to the nearest 10 or 100.
- 3.NBT.2** Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.

- 3.MD.1** Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.
- 3.MD.2** Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). (Excludes compound units such as cm³ and finding the geometric volume of a container.) Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem. (Excludes multiplicative comparison problems, i.e., problems involving notions of “times as much”; see Glossary, Table 2.)

Focus Standards for Mathematical Practice

¹ 3.NBT.3 is taught in Module 3.

- MP.2 Reason abstractly or quantitatively.** Students decontextualize metric measurements and time intervals in minutes as they solve problems involving addition, subtraction, and multiplication. They round to estimate and then precisely solve, evaluating solutions with reference to units and with respect to real world contexts.
- MP.4 Model with mathematics.** Students model measurements on the place value chart. They create drawings and diagrams and write equations to model and solve word problems involving metric units and intervals of time in minutes.
- MP.6 Attend to precision.** Students round to estimate sums and differences and then use the standard algorithms for addition and subtraction to calculate. They reason about the precision of their solutions by comparing estimations with calculations, and are attentive to specifying units of measure.
- MP.7 Look for and make use of structure.** Students model measurements on the place value chart. Through modeling they relate different units of measure and analyze the multiplicative relationship of the base ten system.

WIDA Standards

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Explicit instruction for time vocabulary.
- The use of visual tools such as analog clocks and digital clocks, tiles, geoboards, etc.
- Time interval number labels, including fractional partitions.

Student Learning Experiences / Tasks:

Engage NY Module 2

- Warm Up Activities
- Sprints
- Problem Sheets
- Number Bond Dash
- Lesson Activities
- RDW or Read, Draw, Write
- Problem Sets
- Exit Tickets
- Homework

Everyday Math

- Math Games

Critical Terms:

second
meter
centimeter
gram
kilogram
capacity
liquid volume
liter
milliliter
place value
whole number
elapsed time
add (addition)
sum
addend
subtract (subtraction)
difference
estimate
round
strategies
plot
point
about (ELL)
close to (ELL)

Supplemental Terms:

analog clock
quarter to/'till
quarter of
quarter past
quarter after
midnight
noon
compose
continuous
number line
endpoint
interval
horizontal
vertical
halfway
associative
commutative
distributive
standard algorithm

Instructional Resources:

Engage NY <https://www.engageny.org/resource/grade-3-mathematics-module-2>

Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

<http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm>

U46 Curriculum
U-46 Third Grade Mathematics

Module 3 – Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

Domain(s): Operations and Algebraic Thinking

Trimester(s): 1-2

Transfer: *Students will apply...*

1. Knowledge of multiplication and division to solve real-world problems.
2. Problem-solving skills to find the unknown in both multiplication and division problems.
3. Properties of operations to identify arithmetic patterns.
4. The use of equal-sized groups and arrays models to multiplication and division situations.

Equal groups Unknown products problem situation example: There are 7 bags with 3 apples in each bag for the field trip. How many apples in all?

$7 \times 3 = ?$

Understandings: *Students will understand that...*

1. Visual images and numerical patterns of multiplication and division will assist in solving problems.
2. The Properties of Operations will help in performing computation as well as in problem-solving situations. (Distributive Property of Multiplication, Commutative Property of Multiplication, Identity, and Zero.)
3. Metric measurement units are related to place value concepts/multiples of 10.

Essential Question(s):

1. How do modeling multiplication and division problems help in finding solutions?
2. How can the strategy of breaking apart (decomposing) numbers make multiplication easier to understand?
3. How can we use multiplication to solve division problems?
4. How do multiples and factors relate to multiplication and division?
5. What are the Properties of Operations?
6. How does metric measurement connect to multiples of 10?

Knowledge: *Students will know...*

1. Multiplication and division facts.
2. Problem-solving structures for arrays and for equal groups.

Skill: Students will be able to do...

- Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3)
- Determine the unknown whole number in a multiplication and division equation relating three whole numbers. (3.OA.4)
- Apply properties of operations as strategies to multiply and divide. (3.OA.5)
- Understand division as an unknown-factor problem. (3.OA.6)
- Fluently multiply and divide within 100, using various strategies. (3.OA.7)
- Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (3.OA.8)
- Identify arithmetic patterns, and explain them using properties of operations. (3.OA.9)
- Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations. (3.NBT.3)

Clusters/Standards:

Focus Grade Level Standards

Represent and solve problems involving multiplication and division.¹

3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem. (See Glossary, Table 2.)

3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$.*

Understand properties of multiplication and the relationship between multiplication and division.²

3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

¹ The balance of this cluster is addressed in Module 1.

² The balance of this cluster is addressed in Module 1.

Multiply and divide within 100.³

- 3.OA.7** Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.⁴

- 3.OA.8** Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
- 3.OA.9** Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Use place value understanding and properties of operations to perform multi-digit arithmetic. (A range of algorithms may be used.)⁵

- 3.NBT.3** Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students engage in exploratory lessons to discover and interpret patterns, and apply their observations to solving multi-step word problems involving all four operations.
- MP.3** **Construct viable arguments and critique the reasoning of others.** As students compare solution strategies, they construct arguments and critique the reasoning of their peers. This practice is particularly exemplified in daily Application Problems and problem-solving specific lessons in which students share and explain their work with one another.
- MP.4** **Model with mathematics.** Students use arrays, tape diagrams, and equations to represent word problem situations.
- MP.5** **Use appropriate tools strategically.** Students analyze problems and select the appropriate tools and pathways to solutions. This is particularly evident as students select problem-solving strategies, and use arithmetic properties as simplifying strategies when appropriate.

³ From this point forward, fluency practice with multiplication and division facts is part of the students' on-going experience.

⁴ After being fully taught in Module 3, this standard (as well as 3.OA.3) continues being practiced throughout the remainder of the school year.

⁵ The balance of this cluster is addressed in Module 2.

MP.7 **Look for and make use of structure.** In this module, patterns emerge as tools for problem solving. Students make use of structure as they utilize the distributive property to establish the $9 = 10 - 1$ pattern, for example, or when they check the solution to a fact using units of 9 by making sure the sum of the digits in the product adds up to 9. They make use of the relationship between multiplication and division as they determine unknown factors and interpret the meanings thereof.

WIDA Standards

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Concrete models of multiplication and division processes.
- Anchor Charts highlighting mathematical vocabulary specific to module.
- Repeated verbalization of processes along with an Anchor Chart highlighting terms and steps.

Student Learning Experiences / Tasks:

Engage NY Module 3

- Warm Up Activities
- Sprints
- Problem Sheets
- Number Bond Dash
- Lesson Activities
- RDW or Read, Draw, Write
- Problem Sets
- Exit Tickets
- Homework
-

Everyday Math

- Everyday Math Games

Academic Vocabulary

Critical Terms:

multiplication
decomposing
array
row
column
multiple
product
factor
divide
division
divisor
dividend
quotient
remainder
equal groups
number of groups
size of a group
equal shares
fact family/related facts
distributive property
commutative property
associative property

Supplemental Terms:

inverse operation
zero property
identity
equation
parentheses
rotate

Instructional Resources:

Engage NY <https://www.engageny.org/resource/grade-3-mathematics-module-3>
Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

<http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm>

U46 Curriculum
U-46 Third Grade Mathematics

Module 4: Multiplication and Area
Domain(s): Measurement and Data
Trimester(s): 2

Transfer: *Students will apply...*

1. Area measurement to real-world problem solving situations.
2. Add To, Result Unknown Example: If the area of a garden is measured in square feet, one side of the garden is 8 feet and another side is 2 feet. What is the area of the garden? Solution: $8 + 8 = ?$ or $2 + 2 + 2 + 2 = 16$ or $2 \times 8 = 16$
3. Knowledge of multiplication and division to solve real-world problems.
4. Problem-solving skills to find the unknown in both multiplication and division problems.
5. Properties of operations to identify arithmetic patterns.
6. The use of equal-sized groups, arrays, and area models to multiplication situations.
7. Understanding of area models of multiplication.
8. Problem-solving situations to multiply and divide to solve real-world problem situations.

Understandings: *Students will understand that*

1. Everyday objects have a variety of attributes, each of which can be measured in many ways.
2. Area and addition and multiplication are related.
3. Area is additive.
4. Modeling multiplication and division problems based upon their problem-solving structure can help in finding solutions.
5. There is a relationship between area and multiplication.
6. Properties of Operations will assist in problem-solving situations.
7. Visual images and numerical patterns of multiplication will assist in solving problems.
8. The Properties of Operations will help in performing computation as well as in problem-solving situations. (Distributive Property of Multiplication, Commutative Property of Multiplication, Identity, and Zero.)

Essential Question(s):

1. How can understanding the relationship between addition or multiplication and area aid in problem solving?
2. How does modeling multiplication and division problems help in finding solutions?
3. How can the strategy of breaking apart (decomposing) numbers make multiplication easier to understand?

4. How do multiples and factors relate to multiplication?
5. How can modeling multiplication and divisions problems help in finding their solutions?
6. What is the relationship between area and multiplication?

Knowledge: *Students will know...*

1. Multiplication and division facts.
2. Problem-solving structures for area/arrays and for equal groups.

Skill: *Students will be able to do...*

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities. (3.OA.3)
Use drawings and equations with a symbol for the unknown number to represent the problem. (3.OA.3)
Relate area to the operations of multiplication and addition. (3.MD.7)
Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a + b$ and $a + c$. (3.MD.7)
Use area models to represent the distributive property in mathematical reasoning. (3.MD.7)
Interpret products of whole numbers as the total number of objects in “so many” groups of “so many” objects each. (3.OA.1)
Apply properties of operations as strategies to multiply. (3.OA.5)

Clusters/Standards:

Focus Grade Level Standards

Geometric Measurement: understand concepts of area and relate area to multiplication and to addition.

- 3.MD.5** Recognize area as an attribute of plane figures and understand concepts of area measurement:
- a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area.
 - b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.
- 3.MD.6** Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
- 3.MD.7** Relate area to the operations of multiplication and addition.
- a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
 - b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and

mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.

- c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and $b + c$ is the sum of $a \times b$ and $a \times c$. Use area models to represent the distributive property in mathematical reasoning.
- d. Recognize area as additive. Find the areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Foundational Standards

- 2.MD.1** Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
- 2.MD.2** Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.
- 2.G.2** Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.

Focus Standards for Mathematical Practice

- MP.2** Reason abstractly and quantitatively. Students build toward abstraction starting with tiling a rectangle, then gradually moving to finishing incomplete grids and drawing grids of their own, then eventually working purely in the abstract, imaging the grid as needed.
- MP.3** Construct viable arguments and critique the reasoning of others. Students explore their conjectures about area by cutting to decompose rectangles and then recomposing them in different ways to determine if different rectangles have the same area. When solving area problems, students learn to justify their reasoning and determine whether they have found all possible solutions, when multiple solutions are possible.
- MP.6** Attend to precision. Students precisely label models and interpret them, recognizing that the unit impacts the amount of space a particular model represents, even though pictures may appear to show equal sized models. They understand why when side lengths are multiplied the result is given in square units.
- MP.7** Look for and make use of structure. Students relate previous knowledge of the commutative and distributive properties to area models. They build from spatial structuring to understanding the number of area-units as the product of number of units in a row and number of rows.
- MP.8** Look for and express regularity in repeated reasoning. Students use increasingly sophisticated strategies to determine area over the course of the module. As they analyze and compare strategies, they eventually realize that area can be found by multiplying the number in each row by the number of rows.

WIDA Standard:

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Concrete models of multiplication and division processes.

- Repeated verbalization of processes along with an Anchor Chart highlighting terms and steps.

Student Learning Experiences / Tasks:

Engage NY Module 4

- Warm Up Activities
- Sprints
- Problem Sheets
- Number Bond Dash
- Lesson Activities
- RDW or Read, Draw, Write
- Problem Sets
- Exit Tickets
- Homework

Everyday Math

- Math Games

Academic Vocabulary

Critical Terms:

Area
Area model
Square unit
Tile
Unit square
Whole number
Array
Commutative property
Distribution
Geometric shape
Length
Multiplication
Rows and columns
decomposing
multiple
product
factor

Supplemental Terms:

equation

Instructional Resources:

Engage NY <https://www.engageny.org/resource/grade-3-mathematics-module-4>

Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

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U46 Curriculum
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Module 5 – Fraction as Numbers on the Number Line

Domain(s): Number and Operations – Fractions/Measurement and Data/Geometry

Trimester(s): 2 and 3rd

Transfer: *Students will apply...*

1. Problem-solving skills to understand fractions as they relate to real-world problem situations, such as in measurement, cooking, pizza, money, music, etc.

Understandings: *Students will understand that...*

1. The size of the fractional part is relative to the size of the whole.
2. Fractions represent quantities where a whole is divided into equal-sized parts using models, manipulatives, words, and/or number lines.
3. Fractions can be used as a tool to understand and model quantities and relationships.
4. Fractions are composed of unit fractions.
5. Fractions that represent equal-sized quantities are equivalent.

Essential Question(s):

1. What do fractions represent?
2. What makes fractions equivalent?

Knowledge: *Students will know...*

1. How to represent and Compare Fractions.
2. How to find Equivalent Fractions.

Skill: *Students will be able to do...*

1. Divide shapes into parts with equal areas. (3.G.2)
2. Represent the area of each part as a unit fraction. (3.G.2)
3. Represent a whole using unit fractions. (3.NF.1)

4. Use the term numerator to indicate the number of parts and denominator to represent the total number of parts a whole is partitioned into. (3.NF.1)
5. Represent a fraction as the composition of unit fractions. (3.NF.1)
6. Divide a number line diagram into equal segments and label the appropriate fractional parts. (3.NF.2)
7. Model equivalent fractions using manipulatives, pictures, or number line diagrams and explain in words why the fractions are equivalent. (3.NF.3)
8. Represent whole numbers as fractions using area models, number line diagrams, and numbers. (3.NF.3)
9. Compare two fractions with the same numerator or same denominator using visual models, symbols and words. (3.NF.3)
10. Recognize that comparisons are valid only when the two fractions refer to identical wholes. (3.NF.3)
11. Generate measurement data by measuring lengths to the $\frac{1}{4}$ and $\frac{1}{2}$ inch. (3.MD.4)
12. Show data in a line plot given a scale in $\frac{1}{2}$, $\frac{1}{4}$, or whole numbers. (3.MD.4)

Clusters/Standards:

Focus Grade Level Standards

Develop understanding of fractions as numbers.

- 3.NF.1** Understand a fraction $\frac{1}{b}$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction $\frac{a}{b}$ as the quantity formed by a parts of size $\frac{1}{b}$.
- 3.NF.2** Understand a fraction as a number on the number line; represent fractions on a number line diagram.
- a. Represent a fraction $\frac{1}{b}$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $\frac{1}{b}$ and that the endpoint of the part based at 0 locates the number $\frac{1}{b}$ on the number line.
 - b. Represent a fraction $\frac{a}{b}$ on a number line diagram by marking off a lengths $\frac{1}{b}$ from 0. Recognize that the resulting interval has size $\frac{a}{b}$ and that its endpoint locates the number $\frac{a}{b}$ on the number line.
- 3.NF.3** Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
- a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.
 - b. Recognize and generate simple equivalent fractions, e.g., $\frac{1}{2} = \frac{2}{4}$, $\frac{4}{6} = \frac{2}{3}$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.
 - c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. *Examples: Express 3 in the form of $3 = \frac{3}{1}$; recognize that $\frac{6}{1} = 6$; locate $\frac{4}{4}$ and 1 at the same point of a number line diagram.*
 - d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$,

=, or <, and justify the conclusions, e.g., by using a visual fraction model.

Reason with shapes and their attributes.¹

- 3.G.2** Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. *For example, partition a shape into 4 parts with equal area and describe the area of each part as $\frac{1}{4}$ of the area of the shape* Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch.

Focus Standards for Mathematical Practice

- MP.2 Reason abstractly and quantitatively.** Students represent fractions concretely, pictorially, and abstractly and move back and forth between representations. Students also represent word problems involving fractions pictorially and then express the answer in the context of the problem.
- MP.3 Construct viable arguments and critique the reasoning of others.** Students reason about the area of a shaded region to decide what fraction of the whole it represents.
- MP.6 Attend to precision.** Students specify the whole amount when referring to a unit fraction and explain what is meant by *equal parts* in their own words.
- MP.7 Look for and make use of structure.** Students understand and use the unit fraction as the basic building block or structure of all fractions on the number line.

WIDA Standards

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Concrete models of multiplication and division processes.
- Anchor Charts and visuals highlighting mathematical vocabulary specific to fractions.

Student Learning Experiences / Tasks:

Engage NY Module 5

- Warm Up Activities
- Sprints
- Problem Sheets
- Number Bond Dash

¹ 3.G.1 is taught in Module 7.

- Lesson Activities
- RDW or Read, Draw, Write
- Problem Sets
- Exit Tickets
- Homework

Everyday Math

- Math Games

Academic Vocabulary:

Critical Terms:	Supplemental Terms:
partition	arrays
equal parts	line plot
fraction	half
equal distance (intervals)	third
equivalent	fourth
equivalence	part – part - whole
reasonable	comparison
denominator	linear measurement (using a unit fraction to show distance)
numerator	
justify	
unit fraction	
sixth	
eighth	

Instructional Resources:

Engage NY <https://www.engageny.org/resource/grade-3-mathematics-module-5>

Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

<http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm>

U46 Curriculum
U-46 3rd Grade Mathematics

Module 6: Collecting and Displaying Data

Domain: Measurement and Data (3.MD)

Trimester: 3rd

Transfer: *Students will apply...*

1. Addition and subtraction problem solving skills to create and interpret picture and bar graphs.
2. Problem-solving skills to understand fractions as they relate to real-world problem situations, such as in measurement, cooking, pizza, money, music, etc.

Understandings: *Students will understand that...*

1. Information can be represented in bar graph and picture graph form. These graphs can be used to help us solve one and two- step math problems.
2. The size of the fractional part is relative to the size of the whole.
3. Fractions represent quantities where a whole is divided into equal-sized parts using models, manipulatives, words, and/or number lines.

Essential Question(s):

1. How do we use data represented in bar graphs and picture graphs to make sense of world around us?
2. What do fractions represent?

Knowledge: *Students will know...*

1. Draw and label a picture graph and bar graph
2. Solve problems using information in a bar graph
3. Generate measurement data by measuring lengths to the $\frac{1}{4}$ and $\frac{1}{2}$ inch.
4. Show data in a line plot given a scale in $\frac{1}{2}$, $\frac{1}{4}$, or whole numbers.

Skill: *Students will be able to do...*

1. Draw and label a picture graph and bar graph to represent a data set (including the scale, title, categories, etc.). (3.MD.3)
2. Solve one- and two-step “how many more” and “how many less” problems using information presented in bar graphs. (3.MD.3)
3. Generate measurement data by measuring lengths to the $\frac{1}{4}$ and $\frac{1}{2}$ inch. (3.MD.4)
4. Show data in a line plot given a scale in $\frac{1}{2}$, $\frac{1}{4}$, or whole numbers. (3.MD.4)

Clusters/Standards:

Focus Grade Level Standards

Represent and interpret data.

3.MD.3 Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two- step “how many more” and “how many less” problems using information presented in scaled bar graphs. *For example, draw a bar graph in which each square in the bar graph might represent 5 pets.*

3.MD.4 Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves, or quarters.

Foundational Standards

2.MD.5 Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units, e.g., by using drawings (such as drawings of rulers) and equations with a symbol for the unknown number to represent the problem.

2.MD.6 Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2, ..., and represent whole-number sums and differences within 100 on a number line diagram.

2.MD.9 Generate measurement data by measuring lengths of several objects to the nearest whole unit, or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.

2.MD.10 Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph. (See CCLS Glossary, Table 2.)

Focus Standards for Mathematical Practice

MP.2 Reason abstractly and quantitatively. Students work with data in the context of science and other content areas and interpret measurement data using line plots. Students decontextualize data to create graphs, then contextualize as they analyze their representations to solve problems.

MP.5 Use appropriate tools strategically. Students create and use rulers marked in inches, half inches, and quarter inches. Students plot measurement data on a line plot. They reason about the appropriateness of a line plot as a tool to display fractional measurements.

MP.6 Attend to precision. Students generate rulers using precise measurements, then measure lengths to the nearest quarter inch to collect and record data. Students label axes on graphs to clarify the relationship between quantities and units. They attend to the scale on the graph to precisely interpret the quantities involved.

MP.7 Look for and make use of structure. Students use an auxiliary line to create equally spaced increments on a six-inch strip, which is familiar from the previous module. Students look for trends in the data to help them solve problems and draw conclusions about the data.

WIDA Standards

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.
English language learners will benefit from:

- Explicit instruction for time and perimeter measurement vocabulary, and picture graph and bar graph vocabulary.
- Vertical and horizontal scaled graph models, including picture graphs and bar graphs.
- Anchor Charts highlighting mathematical vocabulary specific to unit.
- Repeated practice verbalizing solution pathways.

Student Learning Experiences / Tasks:

Engage NY Units

Warm-up Activities

- Sprints/Pattern Sheets/Number Bond Dash

Lesson Activities

- Read, Draw, Writes
- Problem Sets
- Exit Tickets
- Homework

Everyday Math

- Math Games

Academic Vocabulary

Axis (vertical or horizontal scale in a graph)

- Frequent (most common measurement on a line plot)
- Measurement data (e.g., length measurements of a collection of pencils)
- Scaled graphs (bar or picture graph in which the scale uses units with a value greater than 1)
- Survey (collecting data by asking a question and recording responses)

Familiar Terms and Symbols¹

- Bar graph (graph generated from categorical data with bars to represent a quantity)
- Data (information)
- Fraction (numerical quantity that is not a whole number, e.g.,)

- Line plot (display of measurement data on a horizontal line)
- Picture graph (graph generated from categorical data with graphics to represent a quantity)

Instructional Resources:

<https://www.engageny.org/resource/grade-3-mathematics-module-6>

Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

<http://schools.nyc.gov/Academics/CommonCoreLibrary/TasksUnitsStudentWork/default.htm>

U46 Curriculum
U-46 3rd Grade Mathematics

Module 7: Geometry and Measurement Word Problems

Domain: Measurement and Data (3.MD)

Trimester: 3rd

Transfer: *Students will apply...*

1. Problem-solving skills to understand fractions as they relate to real-world problem situations, such as in measurement, cooking, pizza, money, music, etc.
2. Knowledge of perimeter to real-world problem solving situations.
3. Knowledge of shapes by drawing representations in different categories that share attributes and recognize those shapes (quadrilaterals) in real world settings.

Understandings: *Students will understand that...*

1. The size of the fractional part is relative to the size of the whole.
2. Fractions represent quantities where a whole is divided into equal-sized parts using models, manipulatives, words, and/or number lines.
3. Perimeter and addition are related.
4. A linear unit is used to measure perimeter.
5. Objects can be described and compared using their geometric attributes.
6. Figures are categorized according to their attributes.

Essential Question(s):

1. What do fractions represent?
2. How can 2-dimensional shapes be described?
3. How are geometric figures constructed?

Knowledge: *Students will know...*

1. Generate measurement data by measuring lengths to the $\frac{1}{4}$ and $\frac{1}{2}$ inch.
2. Show data in a line plot given a scale in $\frac{1}{2}$, $\frac{1}{4}$, or whole numbers.
3. Geometric shapes that represent quadrilaterals.
4. Shapes are categorized.

5. Quadrilaterals are two-dimensional.

Skill: *Students will be able to do...*

1. Generate measurement data by measuring lengths to the $\frac{1}{4}$ and $\frac{1}{2}$ inch. (3.MD.4)
2. Show data in a line plot given a scale in $\frac{1}{2}$, $\frac{1}{4}$, or whole numbers. (3.MD.4)
3. Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths. (3.MD.8)
4. Solve real world and mathematical problems involving perimeters of polygons, including finding an unknown side length. (3.MD.8)
5. Analyze, compare, and classify 2-dimensional shapes by their properties. (3.G.1)
6. Draw shapes with pre-specified attributes. (3.G.1)
7. Investigate, describe, and reason about decomposing and composing quadrilaterals to make other quadrilaterals. (3.G.1)
8. Rotate arrays physically and mentally to view them as compositions of smaller arrays. (3.G.1)
9. Understand that shapes in different categories may share attributes and belong to a larger category. (3.G.1)
10. Recognize and draw examples of more complex quadrilaterals. (3.G.1)

Clusters/Standards:

Focus Grade Level Standards

Represent and interpret data.

- 3.MD.4** Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units – whole numbers, halves or quarters.

Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.

- 3.MD.8** Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

Reason with shapes and their attributes.

- 3.G.1** Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.

Focus Standards for Mathematical Practice

- MP.1** **Make sense of problems and persevere in solving them.** Students demonstrate their ability to persevere by drawing shapes with pre-specified attributes.
- MP.2** **Reason abstractly and quantitatively.** Students demonstrate reasoning by justifying and explaining attributes of quadrilaterals in words and drawings.
- MP.3** **Construct viable arguments and critique the reasoning of others.** Students will be able to explain why specific shapes are called quadrilaterals. They will also listen to each other and explain what their peers have said.
- MP.4** **Model with mathematics.** In this unit, students are asked to use various modalities and model shapes with manipulatives or drawings. They are asked to communicate how their visuals represent these shapes.
- MP.5** **Use appropriate tools strategically.** Students will use concrete models to represent shapes. Students will use concrete models to represent shapes.
- MP.6** **Attend to precision.** Students precisely solve problems such as finding all the possible different compositions of quadrilaterals that make other quadrilaterals.
- MP.7** **Look for and make use of structure.** Students notice structure when they represent quantities by using drawings and equations to represent the commutative and distributive properties. The relationship between multiplication and division also highlights structure for students as they determine the unknown whole number in a multiplication or division statement.
- MP.8** **Look for and express regularity in repeated reasoning.** Students will notice commonalities in attributes.

WIDA Standards

English language learners communicate information, ideas and concepts necessary for academic success in the content area of Mathematics.

English language learners will benefit from:

- Labeling shapes on anchor charts and providing manipulatives in those shapes for identification/matching purposes.
- Identifying real world objects that represent various quadrilaterals.
- Relating the terms 'quad' and 'lateral' to students' native languages.

Student Learning Experiences / Tasks:

Engage NY Units

Warm-up Activities

- Sprints/Pattern Sheets/Number Bond Dash

Lesson Activities

- Read, Draw, Writes
- Problem Sets
- Exit Tickets
- Homework

Everyday Math

- Math Games

Academic Vocabulary:

Critical Terms: Supplemental Terms:

Quadrilateral	Angle
Rhombus	Degree
Rectangle	Compare
Square	Flat
Attribute	Solid
Geometric	3-dimensional
2-dimensional	
Plane	

Instructional Resources:

<https://www.engageny.org/resource/grade-3-mathematics-module-7>

Everyday Math Games

Assessments: Mid-module and end of module assessments can be found by following the above link. The performance tasks can be used as an alternate assessment.

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