

## Exponents and Exponential Functions

### Investigate rational exponents (7.3)

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Rewrite expressions with exponents</b> (N.RN.2)	Can extend thinking beyond the standard, including tasks that may involve one of the following: <ul style="list-style-type: none"> <li>• Designing</li> <li>• Connecting</li> </ul>	Convert between rational exponents and radical expressions  Use the properties of exponents to rewrite (simplify) radical expressions	Convert between rational exponents and radical expressions  Use the properties of exponents to rewrite (simplify) radical expressions ( <b>limited to square roots and cube roots</b> )	Convert between rational exponents and radical expressions	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1
<b>Explain rational exponents</b> (N.RN.1)	<ul style="list-style-type: none"> <li>• Synthesizing</li> <li>• Applying</li> <li>• Justifying</li> <li>• Critiquing</li> <li>• Analyzing</li> <li>• Creating</li> </ul>	<b>Explain</b> how a radical expression can be represented by rational exponents	Show how a radical expression can be represented by <b>rational</b> exponents	Show how an expression can be represented by <b>integer</b> exponents	
<b>Create equivalent expressions</b> (A.SSE.3c)	<ul style="list-style-type: none"> <li>• Proving</li> </ul>	<b>Create</b> equivalent expressions using rational exponents and radical expressions	<b>Create</b> equivalent expressions using rational exponents and radical expressions ( <b>limited to square and cube roots</b> )	<b>Identify</b> equivalent expressions using rational exponents and radical expressions	

N.RN.1 Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{((1/3)3)}$  to hold, so  $(5^{1/3})^3$  must equal 5.

N.RN.2 Rewrite expressions involving radicals and rational exponents using the properties of exponents.

A.SSE.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression  $1.15t$  can be rewritten as  $(1.151/12)^{12t} \approx 1.01212t$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

## Exponents and Exponential Functions

### Represent exponential functions (8.1)

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Identify transformations</b> (F.BF.3)	<p>Can extend thinking beyond the standard, including tasks that may involve one of the following:</p> <ul style="list-style-type: none"> <li>Designing</li> <li>Connecting</li> <li>Synthesizing</li> <li>Applying</li> <li>Justifying</li> <li>Critiquing</li> <li>Analyzing</li> <li>Creating</li> <li>Proving</li> </ul>	<p>Identify the effect on a graph by replacing <math>f(x)</math> with a single transformation:</p> <ul style="list-style-type: none"> <li><math>f(x) + k</math></li> <li><math>k f(x)</math>,</li> <li><math>f(kx)</math></li> <li><math>f(x + k)</math></li> </ul> <p>for specific positive and negative values of <math>k</math></p> <p>Given the graph of a function and a single transformation (<b>for all listed above</b>), find the value of the constant or coefficient</p>	<p>Identify the effect on a graph by replacing <math>f(x)</math> with a single transformation (<b>3 of the 4</b>):</p> <ul style="list-style-type: none"> <li><math>f(x) + k</math></li> <li><math>k f(x)</math>,</li> <li><math>f(kx)</math></li> <li><math>f(x + k)</math></li> </ul> <p>for specific positive and negative values of <math>k</math></p> <p>Given the graph of a function and a single transformation (<b>3 of the 4 listed above</b>), find the value of the constant or coefficient</p>	<p>Identify the effect on a graph by replacing <math>f(x)</math> with a single transformation (<b>2 of the 4</b>):</p> <ul style="list-style-type: none"> <li><math>f(x) + k</math></li> <li><math>k f(x)</math>,</li> <li><math>f(kx)</math></li> <li><math>f(x + k)</math></li> </ul> <p>for specific positive and negative values of <math>k</math></p> <p>Given the graph of a function and a single transformation (<b>2 of the 4 listed above</b>), find the value of the constant or coefficient</p>	<p>Little evidence of reasoning or application to solve the problem</p> <p>Does not meet the criteria in a level 1</p>
<b>Interpret key features</b> (F.IF.4*, F.IF.5)		<p>Identify and interpret <u>all</u> key features in a table <u>and</u> graph in context of the situation.</p> <ul style="list-style-type: none"> <li>intercepts</li> <li>intervals where functions are increasing or decreasing</li> <li>intervals where the function is positive or negative</li> <li>end behavior</li> <li>domain</li> </ul> <p>Translate a verbal description of a relationship to sketch a linear, <u>and</u> exponential graph.</p>	<p>Identify <u>all</u> and interpret <u>at least 3</u> key features from a table <u>or</u> graph in <u>context of the situation</u>.</p> <ul style="list-style-type: none"> <li>intercepts</li> <li>intervals where functions are increasing or decreasing</li> <li>intervals where the function is positive or negative</li> <li>end behavior</li> <li>domain</li> </ul> <p>Translate a verbal description of a graph's key features to <u>sketch</u> a linear <u>or</u> exponential graph.</p>	<p>Identify <u>at least 3</u> key features from a table <u>or</u> graph</p> <ul style="list-style-type: none"> <li>intercepts</li> <li>intervals where functions are increasing or decreasing</li> <li>intervals where the function is positive or negative</li> <li>end behavior</li> <li>domain</li> </ul> <p>Translate a verbal description of a graph's key features to <u>identify</u> a linear <u>or</u> exponential graph.</p>	
<b>Calculate and interpret rate of change</b> (F.IF.6*)		<p>Calculate the average rate of change over a given interval and explain the meaning in context for linear and exponential functions presented in symbolic, table <u>and</u> graph form</p>	<p>Calculate the average rate of change over a given interval <u>and explain the meaning in context</u> for linear and exponential functions presented in symbolic, table <u>or</u> graph form</p>	<p>Calculate the average rate of change over a given interval for linear and exponential functions presented in symbolic, table <u>or</u> graph form</p>	
<b>Combine functions</b> (F.BF.1)		<p>Combine linear, exponential, and quadratic functions <u>to model real world situations</u>.</p>	<p>Combine linear, exponential, <u>and</u> quadratic functions</p>	<p>Combine linear, exponential, <u>or</u> quadratic functions</p>	

- F.BF.1 Write a function that describes a relationship between two quantities.
- Determine an explicit expression, a recursive process or steps for calculation from a context.
  - Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to decaying exponential and relate these functions to the model
- F.BF.3 Identify the effect on the graph of replacing  $f(x)$  by  $f(x + k)$ ,  $k f(x)$ ,  $f(kx)$  and  $f(x) + k$ , for specific values of  $k$  (both negative and positive); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- F.IF.4 For a linear, exponential, or quadratic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. \*
- F.IF.5 Relate the domain of a linear, exponential, ~~or quadratic function~~ to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function. \*
- F.IF.6\* Calculate and interpret the average rate of change of a linear, exponential, ~~or quadratic~~ function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph of a function over a specified interval. \*

## Exponents and Exponential Functions

### Analyze growth and decay models (8.2)

CCSS	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
<b>Interpret key features of exponential growth and decay</b> (F.LE.5*, A.SSE.1b*, F.IF.8b)	Can extend thinking beyond the standard, including tasks that may involve one of the following:	Differentiate between exponential growth and exponential decay  Interpret the growth rate <b>and the growth factor</b> of exponential functions in context of the situation	Differentiate between exponential growth and exponential decay  <b>Interpret</b> the growth rate of exponential functions <b>in context of the situation</b>	Differentiate between exponential growth and exponential decay  Identify the growth rate of exponential functions	Little evidence of reasoning or application to solve the problem  Does not meet the criteria in a level 1
<b>Create and graph equations</b> (A.CED.2*, F.IF.7e)	<ul style="list-style-type: none"> <li>Designing</li> <li>Connecting</li> <li>Synthesizing</li> <li>Applying</li> <li>Justifying</li> <li>Critiquing</li> <li>Analyzing</li> <li>Creating</li> <li>Proving</li> </ul>	Create equations in two or more variables to represent relationships in contextual situations  Graph exponential functions expressed in symbolic form and show key features of the graph ( <b>including labels and scales on the graph</b> )	<b>Create</b> equations in two or more variables to represent relationships in contextual situations  Graph exponential functions expressed in symbolic form <b>and show key features of the graph</b>	<b>Identify</b> equations in two or more variables to represent relationships in contextual situations  Graph exponential functions expressed in symbolic form	
<b>Rewrite and explain expressions</b> (A.SSE.3c)		Use properties of exponents, including rational exponents, to write an equivalent exponential <b>function to reveal and explain specific information</b>	Use properties of exponents, <b>including rational exponents</b> , to write an equivalent exponential function	Use properties of exponents, including rational exponents ( <b>only <math>\frac{1}{2}</math></b> ), to write an equivalent exponential function	
<b>Distinguish between linear and exponential</b> (F.LE.1*)		Explain whether a function is linear or exponential by describing its growth over intervals of equal width when analyzing a table, a graph, <b>and</b> function rule in context of a situation	<b>Explain whether a function is linear or exponential by describing its growth over intervals of equal width</b> when analyzing a table, a graph, <b>or</b> function rule in context of a situation	<b>Recognize</b> a linear or exponential function when analyzing a table, a graph, <b>or</b> function rule, in context of a situation	
<b>Find solutions graphically</b> (A.REI.11)		For linear and/or exponential functions, find intersection points using technology, graphs, and tables and <b>explain in the context of a situation</b>	For linear and/or exponential functions, find intersection points using technology, graphs, <b>and</b> tables	For linear and/or exponential functions, find intersection points using technology, graphs <b>or</b> tables	

A.SSE.1b\* Interpret expressions that represent a quantity in terms of its context.

b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

A.SSE.3c Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.

c. Use the properties of exponents to transform expressions for exponential functions. For example the expression  $1.15t$  can be rewritten as  $(1.151/12)12t \approx 1.01212t$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

A.CED.2\* Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

A.REI.11\* Explain why the x-coordinates of the points where the graphs of the equations  $y = f(x)$  and  $y = g(x)$  intersect are the solutions of the equation  $f(x) = g(x)$ ; find solutions to  $f(x) = g(x)$  approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where  $f(x)$  and/or  $g(x)$  are linear, quadratic, or exponential functions. \*(Modeling Standard)

F.IF.7e Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior, ~~and trigonometric functions, showing period, midline, and amplitude~~

F.IF.8b Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)12^t$ ,  $y = (1.2)^t/10$ , and classify them as representing exponential growth or decay.

F.LE.1\* Distinguish between situations that can be modeled with linear functions and with exponential functions. \*(Modeling Standard)

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.5\* Interpret the parameters in a linear or exponential function in terms of a context. \*(Modeling Standard)