Grade 8 * Module 1

Integer Exponents and Scientific Notation

CCSS – 8.EE.1, 8.EE.3, 8.EE.4

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Lesson 1 – Exponential Notation

Essential Questions:

Examples 1-5

Example 1

$5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 =$ _____

Example 2

$\frac{9}{7} \cdot \frac{9}{7} \cdot \frac{9}{7} \cdot \frac{9}{7}$ _____

Example 3

$\left(-\frac{4}{11}\right)^3 =$ _____

Example 4

$(-2)^6 =$ _____

Example 5

$3.8^4 =$ _____

If n is a positive integer, then: 3^n means $(-2.3)^n$ means $\left(\frac{7}{8}\right)^n$ means	
x^n is called x^n means n is the x is the	

On Your Own

Exercise 1

$\underbrace{4 \times \cdots \times 4}_{7 \text{ times}} =$

Exercise 2

$\underbrace{3.6 \times \cdots \times 3.6}_{\_\_\_\_ \text{ times}} = 3.6^{47}$

Exercise 3

$\underbrace{(-11.63) \times \cdots \times (-11.63)}_{34 \text{ times}} =$

Exercise 4

$\underbrace{12 \times \cdots \times 12}_{\_\_\_\_ \text{times}} = 12^{15}$

Exercise 5

$\underbrace{(-5) \times \cdots \times (-5)}_{10 \text{ times}} =$

Exercise 6

$\underbrace{\frac{7}{2} \times \cdots \times \frac{7}{2}}_{21 \text{ times}} =$

Exercise 7

$\underbrace{(-13) \times \cdots \times (-13)}_{6 \text{ times}} =$

Exercise 8

$\underbrace{\left(-\frac{1}{14}\right) \times \cdots \times \left(-\frac{1}{14}\right)}_{10 \text{ times}} =$

Exercise 9

$\underbrace{x \cdot x \cdots x}_{185 \text{ times}} =$

Exercise 10

$\underbrace{x \cdot x \cdots x}_{\_\_\_\_ \text{times}} = x^n$

On Your Own

Exercise 11

Will these products be positive or negative? How do you know?

$(-1) \bullet (-1) \bullet (-1) = (-1)^3$

$(-1) \bullet (-1) \bullet (-1) \bullet (-1) = (-1)^4$

$\underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{12 \text{ times}} = (-1)^{12}$

$\underbrace{(-1) \times (-1) \times \cdots \times (-1)}_{13 \text{ times}} = (-1)^{13}$

Exercise 12 Is it necessary to do all of the calculations to determine the sign of the product? Why or why not?	
Exercise 13 If ***n*** is a positive even number, then $(-55)^n$ is	
If ***n*** is a positive odd number, then $(-72.4)^n$ is	

Exercise 14

Josie says that

$$\underbrace{(-15) \times \cdots \times (-15)}_{6 \text{ times}} = -15^6.$$

Is she correct? How do you know?

Lesson 1 Summary:

Lesson 1 – Independent Practice

1. Use what you know about exponential notation to complete the expressions below.

$\underbrace{(-5) \times \cdots \times (-5)}_{17 \text{ times}} =$

$\underbrace{3.7 \times \cdots \times 3.7}_{\_\_\_ \text{ times}} = 3.7^{19}$

$\underbrace{7 \times \cdots \times 7}_{\_\_\_ \text{ times}} = 7^{45}$

$\underbrace{6 \times \cdots \times 6}_{4 \text{ times}} =$

$\underbrace{4.3 \times \cdots \times 4.3}_{13 \text{ times}} =$

$\underbrace{(-1.1) \times \cdots \times (-1.1)}_{9 \text{ times}} =$

$\underbrace{\left(\frac{2}{3}\right) \times \cdots \times \left(\frac{2}{3}\right)}_{19 \text{ times}} =$

$\underbrace{\left(-\frac{11}{5}\right) \times \cdots \times \left(-\frac{11}{5}\right)}_{\_\_\_ \text{ times}} = \left(-\frac{11}{5}\right)^{x}$

$\underbrace{(-12) \times \cdots \times (-12)}_{\_\_\_ \text{ times}} = (-12)^{15}$

$\underbrace{a \times \cdots \times a}_{m \text{ times}} =$

2. Write an expression with (-1) as its base that will produce a positive product.

3. Write an expression with (-1) as its base that will produce a negative product.

4. Rewrite each number in exponential notation using 2 as the base.

8 = 16 = 32 =

64 = 128 = 256 =

5. Tim wrote 16 as $(-2)^4$. Is he correct?

6. Could -2 be used as a base to rewrite 32? 64? Why or why not?

Lesson 2 – Multiplication of Number in Exponential Form

Essential Questions:

Discussion: How do you multiply powers of the same number?

$3^5 \cdot 3^7 =$	
Examples 1 & 2 $5^2 \cdot 5^4 =$	
$(-\frac{2}{3})^4 \cdot (-\frac{2}{3})^5 =$	
If x is any number and m, n are positive integers, then	$x^m \cdot x^n =$ _____ because

On Your Own

Exercise 1

$\mathbf{14^{23} \cdot 14^8} =$

Exercise 2

$(-72)^{10} \cdot (-72)^{13} =$

Exercise 3

$\mathbf{5^{94} \cdot 5^{78}} =$

Exercise 4

$\mathbf{(-3)^9 \cdot (-3)^5} =$

Exercise 5

Let a be a number.

$a^{23} \cdot a^8 =$

Exercise 6

Let f be a number.

$f^{10} \cdot f^{13} =$

Exercise 7

Let b be a number.

$\boldsymbol{b^{94} \cdot b^{78}} =$

Exercise 8

Let x be a positive integer. If $(-3)^9 \times (-3)^x = (-3)^{14}$, what is x?

What would happen if there were more terms with the same base? Write an equivalent expression for each problem.

Exercise 9

$9^4 \cdot 9^6 \cdot 9^{13} =$

Exercise 10

$2^3 \cdot 2^5 \cdot 2^7 \cdot 2^9 =$

Can the following expressions be simplified? If so, write an equivalent expression. If not, explain why not.

Exercise 11

$6^5 \cdot 4^9 \cdot 4^3 \cdot 6^{14} =$

Exercise 14

$2^4 \cdot 8^2 = 2^4 \cdot 2^6 =$

Exercise 12

$(-4)^2 \cdot 17^5 \cdot (-4)^3 \cdot 17^7 =$

Exercise 15

$3^7 \cdot 9 = 3^7 \cdot 3^2 =$

Exercise 13

$15^2 \cdot 7^2 \cdot 15 \cdot 7^4 =$

Exercise 16

$5^4 \cdot 2^{11} =$

Exercise 17

Let x be a number. Simplify the expression of the following number:

$(2x^3)(17x^7) =$

Exercise 18

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$a(a + b) =$

Exercise 19

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$b(a+b) =$

Exercise 20

Let a and b be numbers. Use the distributive law to simplify the expression of the following number:

$(a+b)(a+b) =$

Discussion

How do you divide different powers of a number x?

$\frac{3^7}{3^5} =$	
If x is any number and m, n are positive integers, then	$\frac{x^m}{x^n} =$_____
Examples 3 & 4 $\frac{\left(\frac{3}{5}\right)^8}{\left(\frac{3}{5}\right)^6} =$	
$\frac{4^5}{4^2} =$	

On Your Own

Exercise 21

$\frac{7^9}{7^6} =$

Exercise 22

$\frac{(-5)^{16}}{(-5)^7} =$

Exercise 23

$\frac{\left(\frac{8}{5}\right)^9}{\left(\frac{8}{5}\right)^2} =$

Exercise 24

$\frac{13^5}{13^4} =$

Exercise 25

Let a, b be nonzero numbers. What is the following number?

$\frac{\left(\frac{a}{b}\right)^9}{\left(\frac{a}{b}\right)^2} =$

Exercise 26

Let x be a nonzero number. What is the following number?

$\frac{x^5}{x^4} =$

Can the following expressions be simplified? If yes, write an equivalent expression for each problem. If not, explain why not.

Exercise 27

$\frac{2^7}{4^2} = \frac{2^7}{2^4} =$

Exercise 28

$\frac{3^{23}}{27} = \frac{3^{23}}{3^3} =$

Exercise 29

$\frac{3^5 \cdot 2^8}{3^2 \cdot 2^3} =$

Exercise 30

$\frac{(-2)^7 \cdot 95^5}{(-2)^5 \cdot 95^4} =$

Exercise 31

Let x be a number. Simplify the expression of each of the following numbers:

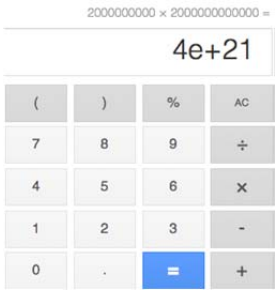
a. $\frac{5}{x^3}(3x^8) =$

b. $\frac{5}{x^3}(-4x^6) =$

c. $\frac{5}{x^3}(11x^4) =$

Exercise 32

Anne used the calculator on her computer to multiply $2{,}000{,}000{,}000 \times 2,000,000,000,000.$ The answer showed up on the calculator as $4e+21$, as shown below. Is the answer on the calculator correct? How do you know?



Lesson 2 Summary:

Lesson 2 – Independent Practice

1. A certain ball is dropped from a height of x feet. It always bounces up to $\frac{2}{3}x$ feet. Suppose the ball is dropped from 10 feet and is caught exactly when it touches the ground after the 30^{th} bounce. What is the total distance traveled by the ball? Express your answer in exponential notation.

Bounce	Computation of Distance Traveled in Previous Bounce	Total Distance Traveled (in feet)
1		
2		
3		
4		
30		
n		

2. If the same ball is dropped from 10 feet and is caught exactly at the highest point after the 25^{th} bounce, what is the total distance traveled by the ball? Use what you learned from the last problem.

3. Let a and b be numbers and $b \neq 0$, and let m and n be positive integers. Simplify each of the following expressions as much as possible:

$(-19)^5 \cdot (-19)^{11} =$	$2.7^5 \times 2.7^3 =$
$\frac{7^{10}}{7^3} =$	$\left(\frac{1}{5}\right)^2 \cdot \left(\frac{1}{5}\right)^{15} =$
$\left(-\frac{9}{7}\right)^m \cdot \left(-\frac{9}{7}\right)^n =$	$\frac{ab^3}{b^2} =$

4. Let the dimensions of a rectangle be $(4 \times (871209)^5 + 3 \times 49762105)$ ft. by $(7 \times (871209)^3 - (49762105)^4)$ ft. Determine the area of the rectangle. No need to expand all the powers.

5. A rectangular area of land is being sold off in smaller pieces. The total area of the land is 2^{15} square miles. The pieces being sold are 8^3 square miles in size. How many smaller pieces of land can be sold at the stated size? Compute the actual number of pieces.

Lesson 3 – Numbers in Exponential Form Raised to a Power

Essential Questions:

Discussion

How can we rewrite this expression using multiplication?

(3 + 3 + 3 + 3) + (3 + 3 + 3 + 3) + (3 + 3 + 3 + 3) + (3 + 3 + 3 + 3) + (3 + 3 + 3 + 3)

How can we rewrite this expression using exponents?

(3 • 3 • 3 • 3) x (3 • 3 • 3 • 3) x (3 • 3 • 3 • 3) x (3 • 3 • 3 • 3) x (3 • 3 • 3 • 3)

Examples 1-3

Example 1

$(3^4)^5$ =

Example 2

$(7^2)^6$ =

Example 3

$(1.3^3)^{10}$ =

For any number x and any positive numbers m and n,	$(x^m)^n$ =_____ because

On Your Own

Exercise 1

$(15^3)^9 =$

Exercise 2

$((-2)^5)^8 =$

Exercise 3

$(3.4^{17})^4 =$

Exercise 4

Let s be a number.

$(s^{17})^4 =$

Exercise 5

Sarah wrote $(3^5)^7 = 3^{12}$. Correct her mistake. Write an exponential expression using a base of 3 and exponents of 5, 7, and 12 that would make her answer correct.

Exercise 6

A number y satisfies $y^{24} - 256 = 0$. What equation does the number $x = y^4$ satisfy?

For any numbers x and y and any positive number n,	$(xy)^n$ =_____ because

On Your Own

Exercise 7

$(11 \cdot 4)^9 =$

Exercise 8

$(3^2 \cdot 7^4)^5 =$

Exercise 9

Let a, b, and c be numbers.

$(3^2 a^4)^5 =$

Exercise 10

Let x be a number.

$(5x)^7 =$

Exercise 11

Let x and y be numbers.

$(5xy^2)^7 =$

Exercise 12

Let a, b, and c be numbers.

$(a^2 bc^3)^4 =$

Exercise 13

Let x and y be numbers, $y \neq 0$, and let n be a positive integer. How is $\left(\frac{x}{y}\right)^n$ related to x^n and y^n?

Lesson 3 Summary:

Lesson 3 – Independent Practice

1. Show (prove) in detail why $(2 \cdot 3 \cdot 7)^4 = 2^4 3^4 7^4$.

2. Show (prove) in detail why $(xyz)^4 = x^4 y^4 z^4$ for any numbers x, y, z.

3. Show (prove) in detail why $(xyz)^n = x^n y^n z^n$ for any numbers, y, and z and for any positive integer n.

Lesson 4 – Numbers Raised to the Zeroth Power

Essential Questions:

Exponent Laws	$x^m \cdot x^n =$ $(x^m)^n =$ $(xy)^n =$ $\frac{x^m}{x^n} =$ $\left(\frac{x}{y}\right)^n =$

Exploratory Challenge

What does it mean to raise a positive number x to the zeroth power?
For example, what should 3^0 mean?

For any positive number x,	x^0 = _____

Exploratory Challenge 2

Prove your definition of x^0 = _____ in all of the laws

$x^m \cdot x^n = x^{m+n}$ When m is positive and n = 0	
$x^m \cdot x^n = x^{m+n}$ When m = 0 and n is positive	
$x^m \cdot x^n = x^{m+n}$ When m = 0 and n = 0	

$(x^m)^n = x^{mn}$ When m is positive and n = 0	
$(x^m)^n = x^{mn}$ When m = 0 and n is positive	
$(x^m)^n = x^{mn}$ When m = 0 and n = 0	
$(xy)^n = x^n y^n$ When m is positive and n = 0	
$(xy)^n = x^n y^n$ When m = 0 and n is positive	
$(xy)^n = x^n y^n$ When m = 0 and n = 0	

Exploratory Challenge 3

Write numbers in expanded form

Write 8,374 in expanded form	**8 × _____ + 3 × _____ + 7 × _____ + 4 × _____**
Write the expanded form of $8{,}374$ using exponential notation.	

Write the expanded form of $6{,}985{,}062$ using exponential notation.	

Lesson 4 Summary:

Lesson 4 – Independent Practice

Let x, y be numbers $(x, y \neq 0)$. Simplify each of the following expressions of numbers.

1. $\frac{y^{12}}{y^{12}} =$

2. $9^{15} \cdot \frac{1}{9^{15}} =$

3. $(7(123456.789)^4)^0 =$

4. $2^2 \cdot \frac{1}{2^5} \cdot 2^5 \cdot \frac{1}{2^2} =$

5. $\frac{x^{41}}{y^{15}} \cdot \frac{y^{15}}{x^{41}} =$

Lesson 5 – Negative Exponents and the Laws of Exponents

Essential Questions:

Exponent Laws	$x^m \cdot x^n =$ $(x^m)^n =$ $(xy)^n =$ $\frac{x^m}{x^n} =$ $\left(\frac{x}{y}\right)^n =$

Exploratory Challenge

What does a negative exponent mean? For example, what would 3^{-5} mean?

For any positive number x and integer n,	x^{-n} = _____

On Your Own

Exercise 1

Verify the general statement $x^{-b} = \frac{1}{x^b}$ for $x = 3$ and $b = -5$.

Exercise 2

What is the value of (3×10^{-2})?

Exercise 3

What is the value of (3×10^{-5})?

Exercise 4

Write the complete expanded form of the decimal 4.728 in exponential notation.

For Exercises 5–10, write an equivalent expression, in exponential notation, to the one given and simplify as much as possible.

Exercise 5

$5^{-3} =$

Exercise 6

$\frac{1}{8^9} =$

Exercise 7

$3 \cdot 2^{-4} =$

Exercise 8

Let x be a nonzero number.

$x^{-3} =$

Exercise 9

Let x be a nonzero number.

$\frac{1}{x^9} =$

Exercise 10

Let x, y be two nonzero numbers.

$xy^{-4} =$

Exercise 11

$\frac{19^2}{19^5} =$

Exercise 12

$\frac{\mathbf{17^{16}}}{\mathbf{17^{-3}}} =$

Exercise 13

If we let $b = -1$ in $(x^b)^a = x^{ab}$, a be any integer, and y be any positive number, what do we get?

Exercise 14

Show directly that $\left(\frac{7}{5}\right)^{-4} = \frac{7^{-4}}{5^{-4}}$.

Lesson 5 Summary:

Lesson 5 – Independent Practice

1. Compute: $3^3 \cdot 3^2 \cdot 3^1 \cdot 3^0 \cdot 3^{-1} \cdot 3^{-2} =$

   Compute: $5^2 \cdot 5^{10} \cdot 5^8 \cdot 5^0 \cdot 5^{-10} \cdot 5^{-8} =$

   Compute for a nonzero number, a: $a^m \cdot a^n \cdot a^l \cdot a^{-n} \cdot a^{-m} \cdot a^{-l} \cdot a^0 =$

2. Without using $x^a \cdot x^b = x^{a+b}$, show directly that $(17.6^{-1})^8 = 17.6^{-8}$.

3. Without using $x^a \cdot x^b = x^{a+b}$, show (prove) that for any whole number n and any positive number y, $(y^{-1})^n = y^{-n}$.

4. Show directly without using $\frac{x^m}{x^n} = x^{m-n}$ that $\frac{2.8^{-5}}{2.8^7} = 2.8^{-12}$.

Lesson 6 – Proofs of Laws of Exponents

Essential Questions:

Exponent Laws	$x^a \cdot x^b =$ $(x^b)^a =$ $(xy)^a =$ $\frac{x^a}{x^b} =$ $\left(\frac{x}{y}\right)^a =$

Facts we will use to prove $(x^b)^a = x^{ab}$:

(A) $(x^b)^a = x^{ab}$ is already known to be true when the integers a and b satify a ≥ 0, b ≥ 0.

(B) $x^{-m} = \frac{1}{x^m}$ for any whole number m.

(C) $(\frac{1}{x})^m = \frac{1}{x^m}$ for any whole number m.

Exercise 1

Show that $(\frac{1}{x})^m = \frac{1}{x^m}$ is implied by equation $(\frac{x}{y})^n = \frac{x^n}{y^n}$ when $m > 0$, and explain why $(\frac{1}{x})^m = \frac{1}{x^m}$ continues to hold even when $m = 0$.

Exercise 2

Show that $x^{-m} = \frac{1}{x^m}$ is in fact a special case of $(x^b)^a = x^{ab}$ by rewriting it as $(x^m)^{-1} = x^{(-1)m}$ for any whole number m, so that if $b = m$ (where m is a whole number) and $a = -1$, $(x^b)^a = x^{ab}$ becomes $x^{-m} = \frac{1}{x^m}$

Exercise 3

Show that $(\frac{1}{x})^m = \frac{1}{x^m}$ is a special case of $\left(x^b\right)^a = x^{ab}$ by rewriting $(\frac{1}{x})^m = \frac{1}{x^m}$ as $(x^{-1})^m = x^{m(-1)}$ for any whole number m. Thus, $(\frac{1}{x})^m = \frac{1}{x^m}$ is the special case of $\left(x^b\right)^a = x^{ab}$ when $b = -1$ and $a = m$, where m is a whole number.

Exercise 4

Proof of Case (iii): Show that when $a < 0$ and $b \geq 0$, $(x^b)^a = x^{ab}$ is still valid. Let $a = -c$ for some positive integer c. Show that the left side and right sides of $(x^b)^a = x^{ab}$ are equal.

Lesson 6 Summary:

Lesson 6 – Independent Practice

1. You sent a photo of you and your family on vacation to seven Facebook friends. If each of them sends it to five of their friends, and each of those friends sends it to five of their friends, and those friends send it to five more, how many people (not counting yourself) will see your photo? No friend received the photo twice. Express your answer in exponential notation.

# of New People to View Your Photo	*Total # of People to View Your Photo*

2. Show directly, <u>without</u> using $(x^b)^a = x^{ab}$, that $(1.27^{-36})^{85} = 1.27^{-36 \cdot 85}$.

3. Show directly that $\left(\frac{2}{13}\right)^{-127} \cdot \left(\frac{2}{13}\right)^{-56} = \left(\frac{2}{13}\right)^{-183}$.

4. Prove for any positive number x, $x^{-127} \cdot x^{-56} = x^{-183}$.

5. Prove for any positive number x, $x^{-m} \cdot x^{-n} = x^{-m-n}$ for positive integers m and n.

6. Which of the preceding four problems did you find easiest to do? Explain.

7. Use the properties of exponents to write an equivalent expression that is a product of distinct primes, each raised to an integer power.

$$\frac{10^5 \cdot 9^2}{6^4} =$$

Lesson 7 – Magnitude

Essential Questions:

Fact 1: The number 10^n, for arbitrarily large positive integers n, is a **big number** that given a number M (no matter how big it is) there is a power of 10 that exceeds M.

Fact 2 : The number 10^{-n}, for arbitrarily large positive integers n is a **small number** in the sense that given a positive number S (no matter how small it is), there is a (negative) power of 10 that is smaller than S.

Example 1 Let M be the world population as of March 23, 2013. Approximately, M = 7,073,981,143. How many digits does it have?	
Example 2 Let M be the U.S. national debt on March 23, 2013. M = 16,755,133,009,522 to the nearest dollar. How many digits does it have?	

On Your Own

Exercise 1 Let M = 993,456,789,098,765. Find the smallest power of 10 that will exceed M.	
Exercise 2 Let = $78{,}491\frac{899}{987}$. Find the smallest power of 10 that will exceed M.	

Example 3 – *A General Case of Fact 1* <u>Case 1</u>: Let M be a positive integer with *n* digits.	
Case 2: In Case 1, M was a positive integer.	

On Your Own

Exercise 3 Let M be a positive integer. Explain how to find the smallest power of 10 that exceeds it.	

Example 4 • The average ant weighs about 0.0003 grams. If we want to express this number as a power of 10, how do we go about figuring out what to do?	
• The mass of a neutron of 0.000 000 000 000 000 000 000 000 001 674 9 kilograms What do we need to know to be able to determine the correct power of 10?	

On Your Own

Exercise 4 The chance of you having the same DNA as another person (other than an identical twin) is approximately 1 in 10 trillion (one trillion is a 1 followed by 12 zeros). Given the fraction, express this very small number using a negative power of 10. $\frac{1}{10{,}000{,}000{,}000{,}000}$	

Exercise 5 The chance of winning a big lottery prize is about 10^{-8}, and the chance of being struck by lightning in the U.S. in any given year is about 0.000001. Which do you have a greater chance of experiencing? Explain.	
Exercise 6 There are about 100 million smartphones in the U.S. Your teacher has one smartphone. What share of U.S. smartphones does your teacher have? Express your answer using a negative power of 10.	

Lesson 7 Summary

Lesson 7 – Independent Practice

1. What is the smallest power of 10 that would exceed 987,654,321,098,765,432?

2. What is the smallest power of 10 that would exceed 999,999,999,991?

3. Which number is equivalent to 0.0000001: 10^{7} or 10^{-7}? How do you know?

4. Sarah said that 0.00001 is bigger than 0.001 because the first number has more digits to the right of the decimal point. Is Sarah correct? Explain your thinking using negative powers of 10 and the number line.

5. Place each of the following numbers on a number line in its approximate location:
10^{5} 10^{-99} 10^{-17} 10^{14} 10^{-5} 10^{30}

Lesson 8 – Estimating Quantities

Essential Questions:

Example 1 In 1723, the population of New York City was approximately 7,248. By 1870, almost 150 years later, the population had grown to 942,292.	

On Your Own

Exercise 1 The Federal Reserve states that the average household in January of 2013 had $7,122 in credit card debt. About how many times greater is the U.S. national debt, which is $16,755,133,009,522? Rewrite each number to the nearest power of 10 that exceeds it, and then compare.	Credit Card Debt: National Debt: Compare:

Example 2

Let's compare the population of New York City to the population of New York State. Specifically, let's find out how many times greater the population of New York State is compared to that of New York City. The population of New York City is 8,336,697. Let's round this number to the nearest million; then write it as single-digit integer times a power of 10.

Population of New York City, rounded to the nearest million:

_____

Written as a single-digit integer times a power of 10:

_____

The population of New York State is 19,570,261. Round to the nearest million; then write it as a single-digit integer times a power of 10:

Population of New York State, rounded to the nearest million:

_____

Written as a single-digit integer times a power of 10:

_____

To estimate the difference in size we compare state population to city population:

Example 3

There are about 9 billion devices connected to the Internet. If a wireless router can support 300 devices, about how many wireless routers are necessary to connect all 9 billion devices wirelessly?

On Your Own

Exercise 2

There are about 3,000,000 students attending school, kindergarten through 12th grade, in Illinois.

The average number of students attending a middle school in Illinois is 8×10^2.

How many times greater is the overall number of K-12 students compared to the number of middle school students?

Exercise 3

A conservative estimate of the number of stars in the universe is 6×10^{22}.
The average human can see about 3,000 stars at night with his naked eye.

About how many times more stars are there in the universe, compared to the stars a human can actually see?

Exercise 4 The estimated world population in 2011 was 7×10^9. Of the total population, 682 million of those people were left-handed. Approximately what percentage of the world population is left-handed according to the 2011 estimation?	

Example 4 The average American household spends about $40,000 each year. If there are about 1×10^8 households, what is the total amount of money spent by American households in one year?	

On Your Own

Exercise 5

The average person takes about 30,000 breaths per day.

Express this number as a single-digit integer times a power of 10.

If the average American lives about 80 years (or about 30,000 days), how many total breaths will a person take in her lifetime?

Lesson 8 Summary:

Lesson 8- Independent Practice

1. The Atlantic Ocean region contains approximately 2×10^{16} gallons of water. Lake Ontario has approximately 8,000,000,000,000 gallons of water. How many Lake Ontarios would it take to fill the Atlantic Ocean region in terms of gallons of water?

2. U.S. national forests cover approximately 300,000 square miles. Conservationists want the total square footage of forests to be $300{,}000^2$ square miles. When Ivanna used her phone to do the calculation, her screen showed the following:

a. What does the answer on her screen mean? Explain how you know.

b. Given that the U.S. has approximately 4 million square miles of land, is this a reasonable goal for conservationists? Explain.

3. The average American is responsible for about 20,000 kilograms of carbon emission pollution each year. Express this number as a single-digit integer times a power of 10.

4. The United Kingdom is responsible for about 1×10^4 kilograms. Which country is responsible for greater carbon emission pollution each year? By how much?

Lesson 9 – Scientific Notation

Essential Questions:

Scientific Notation **Order of Magnitude** **Leading Digit**	

Example 1 The finite decimal 234.567 is equal to every one of the following: 2.34567×10^{2} 0.234567×10^{3} 23.4567×10 234.567×10^{0} 234567×10^{-3} 234567000×10^{-6} Which is a representation of 234.567 in scientific notation? Explain how you know the one you chose is correct.	

On Your Own

Are the following numbers written in scientific notation? If not, state why.

Exercise 11.908 X 10^{17}		**Exercise 4** 4.0701 + 10^{7}	
Exercise 2 0.325 X 10^{-2}		**Exercise 5** 18.432 X 5^{8}	
Exercise 3 7.99 X 10^{32}		**Exercise 6** 8 X 10^{-11}	

Consider the following equation in scientific notation:

$$s = d \times 10^{n}$$

What is the exponent n called?

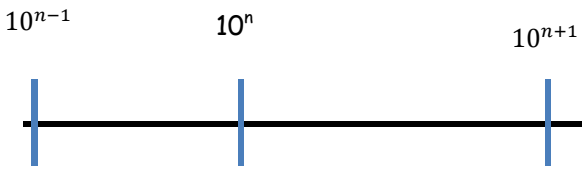
The following inequalities hold:

$10^{n} \leq s$ and $s < 10^{n+1}$

Thus, the exponent n serves to give an approximate location of s on the number line. That is, n gives the approximate magnitude of s.

The inequalities in above can be written as
$10^{n} \leq s < 10^{n+1}$.

Place the number s on the number line.



Example 2

Let's say we need to determine the difference in the populations of Texas and North Dakota. In 2012, Texas had a population of about 26 million people, and North Dakota had a population of about 6.9×10^4. We begin by writing each number in scientific notation:

To find the difference, what operation should we use?

****To compute this easily, we need to make the order of **magnitude** of each number equal. That is, each number must have the **same order of magnitude and the same base.**

Write study questions here that will help you to remember the process and reasoning:

Population of Texas: 26,000,000

In scientific notation:_____

Population of North Dakota: 69,000

In Scientific notation:_____

Example 3 Let's say that we need to find the combined mass of two hydrogen atoms and one oxygen atom, which is normally written as H2O or otherwise known as water. To appreciate the value of scientific notation, the mass of each atom will be given in standard notation: • One hydrogen atom is approximately 0.000 000 000 000 000 000 000 000 001 7 kilograms. • One oxygen atom is approximately 0.000 000 000 000 000 000 000 000 027 kilograms.	
Could we have solved this problem by using a different common order of magnitude?	

On Your Own

Use the table below to complete Exercises 7 and 8.

The table below shows the debt of the three most populous states and the three least populous states.

State	Debt (in dollars)	Population (2012)
California	407,000,000,000	38,000,000
New York	337,000,000,000	19,000,000
Texas	276,000,000,000	26,000,000
North Dakota	4,000,000,000	69,000
Vermont	4,000,000,000	62,600
Wyoming	2,000,000,000	57,600

Exercise 7

a. What is the sum of the debts for the three most populous states? Express your answer in scientific notation.

b. What is the sum of the debt for the three least populous states? Express your answer in scientific notation.

c. How much larger is the combined debt of the three most populous states than that of the three least populous states? Express your answer in scientific notation.	

Exercise 8 a. What is the sum of the population of the three most populous states? Express your answer in scientific notation. b. What is the sum of the population of the three least populous states? Express your answer in scientific notation.	

c. Approximately how many times greater is the total population of California, New York, and Texas compared to the total population of North Dakota, Vermont, and Wyoming?	
Exercise 9 All planets revolve around the sun in elliptical orbits. Uranus's furthest distance from the sun is approximately 3.004×10^9 km, and its closest distance is approximately 2.749×10^9 km. Using this information, what is the average distance of Uranus from the sun?	

Summary – Lesson 9

Lesson 9 – Independent Practice

1. Write the number 68,127,000,000,000,000 in scientific notation. Which of the two representations of this number do you prefer? Explain

2. Here are the masses of the so-called inner planets of the Solar System.

Mercury: 3.3022×10^{23} kg

Earth: 5.9722×10^{24} kg

Venus: 4.8685×10^{24} kg

Mars: 6.4185×10^{23} kg

What is the average mass of all four inner planets? Write your answer in scientific notation.

Lesson 10 – Operations with Numbers in Scientific Notation

Essential Questions:

Example 1 The world population is about 7 billion. There are 4.6×10^7 ants for every human on the planet. About how many ants are there in the world?	
Example 2 A certain social media company processes about 990 billion "likes" per year. If the company has approximately 8.9×10^8 users of the social media, about how many "likes" is each user responsible for per year?	

Exercise 1 The speed of light is 300,000,000 meters per second. The sun is approximately 1.5×10^{11} meters from Earth. How many seconds does it take for sunlight to reach Earth?	
Exercise 2 The mass of the moon is about 7.3×10^{22} kg. It would take approximately 26,000,000 moons to equal the mass of the sun. Determine the mass of the sun.	

Example 3 In 2010, Americans generated 2.5×10^8 tons of garbage. There are about 2,000 landfills in the United States. Assuming that each landfill is the same size and that trash is divided equally among them, determine how many tons of garbage were sent to each landfill in 2010.	

On Your Own

Exercise 3 The mass of Earth is 5.9×10^{24} kg. The mass of Pluto is 13,000,000,000,000,000,000,000 kg. Compared to Pluto, how much greater is Earth's mass than Pluto's mass?	
Exercise 4 Using the information in Exercises 2 and 3, find the combined mass of the moon, Earth, and Pluto.	
Exercise 5 How many combined moon, Earth, and Pluto masses (i.e., the answer to Exercise 4) are needed to equal the mass of the sun (i.e., the answer to Exercise 2)?	

Lesson 10 Summary:

Lesson 10 – Independent Practice

1. The sun produces 3.8×10^{27} joules of energy per second. How much energy is produced in a year? (Note: a year is approximately 31,000,000 seconds).

2. On average, Mercury is about 57,000,000 km from the sun, whereas Neptune is about 4.5×10^{9} km from the sun. What is the difference between Mercury's and Neptune's distances from the sun?

3. The mass of Earth is approximately 5.9×10^{24} kg, and the mass of Venus is approximately 4.9×10^{24} kg.

a. Find their combined mass.

b. Given that the mass of the sun is approximately 1.9×10^{30} kg, how many Venuses and Earths would it take to equal the mass of the sun?

Lesson 11 – Efficacy of Scientific Notation

Essential Questions:

Exercise 1

The mass of a proton is 0.000000000000000000000000001672622 kg.	In scientific notation

Exercise 2

The mass of an electron is 0.000000000000000000000000000910938291 kg.	In scientific notation:
The mass of the proton 0.000000000000000000000000001672622 kg.	In scientific notation

Exercise 3

Write the ratio that compares the mass of a proton to the mass of an electron.	
Example 1 The advantage of the scientific notation becomes even more pronounced when we have to compute how many times heavier a proton is than an electron. Instead of writing the ratio, r, as $$r = \frac{0.000000000000000000000000001672622}{0.000000000000000000000000000910938291}$$	

On Your Own

Exercise 4

Compute how many times heavier a proton is than an electron (that is, find the value of the ratio). Round your final answer to the nearest one.	
Example 2 As of March 23, 2013, the U.S. national debt was $16,755,133,009,522 (rounded to the nearest dollar). According to the 2012 U.S. census, there are about 313,914,040 American citizens. What is each citizen's approximate share of the debt?	

Own Your Own

Exercise 5

The geographic area of California is 163,696 sq. mi., and the geographic area of the U.S. is 3,794,101 sq. mi. Let's round off these figures to 1.637×10^5 and 3.794×10^6. In terms of area, roughly estimate how many Californias would make up one U.S. Then compute the answer to the nearest ones.

Exercise 6

The average distance from Earth to the moon is about 3.84×10^5 km, and the distance from Earth to Mars is approximately 9.24×10^7 km in year 2014. On this simplistic level, how much further is traveling from Earth to Mars than from Earth to the moon?

Lesson 11 Summary:

Lesson 11 – Independent Practice

1. There are approximately 7.5×10^{18} grains of sand on Earth. There are approximately 7×10^{27} atoms in an average human body. Are there more grains of sand on Earth or atoms in an average human body? How do you know?

2. About how many times more atoms are in a human body compared to grains of sand on Earth?

3. Suppose the geographic areas of California and the US are 1.637×10^5 and 3.794×10^6 sq. mi., respectively. California's population (as of 2012) is approximately 3.804×10^7 people. If population were proportional to area, what would be the U.S. population?

4. The actual population of the U.S. (as of 2012) is approximately 3.14×10^8. How does the population density of California (i.e., the number of people per sq. mi.) compare with the population density of the U.S.?

Lesson 12 – Choice of Unit

Essential Questions:

Discussion:	

Discussion example Suppose we want to measure the horizontal dimensions of a dining table. In this case, measurements of 42 × 60 sq. in, or for that matter, $3\frac{1}{2}$ × 5 sq. ft. are commonly accepted. However, What happens when the same measurement is presented as: $\frac{0.7}{1056} X \frac{1}{1056} sq.mi.$.	

On Your Own

Exercise 1 A certain brand of MP3 player will display how long it will take to play through its entire music library. If the maximum number of songs the MP3 player can hold is 1,000 (and the average song length is 4 minutes), would you want the time displayed in terms of seconds-, days-, or years-worth of music? Explain.	
Exercise 2 You have been asked to make frosted cupcakes to sell at a school fundraiser. Each frosted cupcake contains about 20 grams of sugar. Bake sale coordinators expect 500 people will attend the event. Assume everyone who attends will buy a cupcake; does it make sense to buy sugar in grams, pounds, or tons? Explain.	
Exercise 3 The seafloor spreads at a rate of approximately 10 cm per year. If you were to collect data on the spread of the seafloor each week, which unit should you use to record your data? Explain.	

Example 1

In the previous lesson we worked with the masses of protons and electrons, which are:

1.672622×10^{-27} and $9.109382291 \times 10^{-31}$ respectively

What do the factors 10^{-27} and 10^{-31} suggest?

Gigaelectronvolt, $\frac{GeV}{c^2}$, is what particle physicists use as the unit of mass.

1 Gigaelectronvolt = 1.783×10^{-27} kg

Mass of 1 proton = $1.672\,622 \times 10^{-27}$ kg

Exercise 4

Show that the mass of a proton is $0.938 \frac{GeV}{c^2}$.

In popular science writing, a commonly used unit is the light-year, or the distance light travels in one year (note: one year is defined as 365.25 days).

$$1\ light-year = 9,460,730,472,580.800\ km \approx 9.46073 \times 10^{12}\ km$$

Discussion

Example 2

Distances involved in astronomy are very, very large (e.g., astronomical distances). The distance from the sun to the nearest star (Proxima Centauri) is approximately

4.013 336 473 X 10^{13} km.

In 1838, F.W. Bessel2 was the first to measure the distance to a star, 61 Cygni, and its distance from the sun was

1.078 807 X 10^{14} km

On Your Own

Exercise 5

The distance of the nearest star (Proxima Centauri) to the sun is approximately

4.013 336 473 × 10^{13} km.

Show that Proxima Centauri is 4.2421 light-years from the sun.

Exploratory Challenge 1

Finally, let us look at an example involving the masses of the planets in our solar system. They range from Mercury's 3.3022×10^{23} kg to Jupiter's 1.8986×10^{27} kg. However, Earth's mass is the fourth heaviest among the eight planets , and it seems reasonable to use it as the point of reference for discussions among planets. Therefore, a new unit is M_{E}, the mass of the Earth, or 5.97219×10^{24} kg.

Mercury: Let x represent the mass of Mercury in the unit ME. We want to determine what number times the new unit is equal to the mass of Mercury in kg. Since $ME = 5.97219 \times 10^{24}$, then:

$$(5.97219 \text{ X } 10^{24})x = 3.3022 \text{ X } 10^{23}$$

Jupiter:

Let x represent the mass of Jupiter in the unit M_{E}. We want to determine what number times the new unit is equal to the mass of Jupiter in kg. Since $M_{\mathrm{E}} = 5.97219 \times 10^{24}$, then:

$$(5.97219 \text{ X } 10^{24})x = 1.8986 \text{ X } 10^{27}$$

Exploratory Challenge 2

Suppose you are researching atomic diameters and find that credible sources provided the diameters of five different atoms as shown in the table below. All measurements are in cm.

1×10^{-8}	1×10^{-12}	5×10^{-8}	5×10^{-10}	5.29×10^{-11}

Exercise 6

What new unit might you introduce in order to discuss the differences in diameter measurements?

Exercise 7

Name your unit and explain why you chose it.

Exercise 8

Using the unit you have defined, rewrite the five diameter measurements.

Lesson 12 Summary:

Lesson 12 – Independent Practice

1. Verify the claim that, in terms of gigaelectronvolts, the mass of an electron is 0.000511.

2. The maximum distance between Earth and the sun is 1.52098232×10^8 km, and the minimum distance is 1.47098290×10^8 km. What is the average distance between Earth, and the sun in scientific notation?

3. Suppose you measure the following masses in terms of kilograms:

2.6×10^{21}	9.04×10^{23}
8.82×10^{23}	2.3×10^{18}
1.8×10^{12}	2.103×10^{22}
8.1×10^{20}	6.23×10^{18}
6.723×10^{19}	1.15×10^{20}
7.07×10^{21}	7.210×10^{29}
5.11×10^{25}	7.35×10^{24}
7.8×10^{19}	5.82×10^{26}

What new unit might you introduce in order to aid discussion of the masses in this problem? Name your unit and express it using some power of 10. Rewrite each number using your newly defined unit.

Lesson 13 – Comparison of Numbers Written in Scientific Notation and Interpreting Scientific Notation Using Technology

Essential Questions:

What are the basic facts to remember when comparing numbers in scientific notation?	
Example 1 Among the galaxies closest to Earth, M82 is about 1.15×10^7 light-years away, and Leo I Dwarf is about 8.2×10^5 light-years away. Which is closer?	

Exercise 1 The Fornax Dwarf galaxy is 4.6×10^5 light-years away from Earth, while Andromeda I is 2.430×10^6 light-years away from Earth. Which is closer to Earth?	
Example 2 Many subatomic particles are unstable: charged pions have an average lifetime of 2.603×10^{-8} seconds, while muons have an average lifetime of 2.197×10^{-6} seconds. Which has a longer average lifetime?	
Exercise 2 The average lifetime of the tau lepton is 2.906×10^{-13} seconds, and the average lifetime of the neutral pion is 8.4×10^{-17} seconds. Explain which subatomic particle has a longer average lifetime.	

Exploratory Challenge 1/Exercise 3 Theorem: Given two numbers in scientific notation, $a \times 10^m$ and $b \times 10^m$, if $m < n$, then $a \times 10^m < b \times 10^n$. Prove the theorem.	
Exercise 4 Compare 9.3×10^{28} and 9.2879×10^{28}.	

Exercise 5

Chris said that

$5.3 \times 10^{41} < 5.301 \times 10^{41}$

because 5.3 has fewer digits than 5.301.

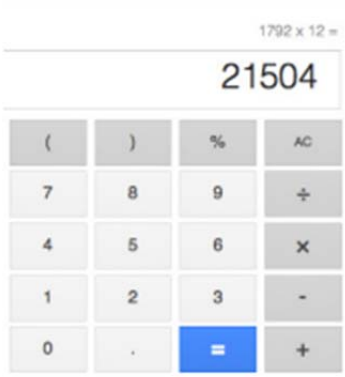
Show that even though his answer is correct, his reasoning is flawed.

Show him an example to illustrate that his reasoning would result in an incorrect answer. Explain.

Exploratory Challenge 2/Exercise 6

You have been asked to determine the exact number of Google searches that are made each year. The only information you are provided is that there are 35,939,938,877 searches performed each week. Assuming the exact same number of searches is performed each week for the 52 weeks in a year, how many total searches will have been performed in one year? Your calculator does not display enough digits to get the exact answer. Therefore, you must break down the problem into smaller parts. Remember, you cannot approximate an answer because you need to find an exact answer. Use the screen shots below to help you reach your answer.

Yahoo is another popular search engine. Yahoo receives requests for 1,792,671,355 searches each month. Assuming the same number of searches is performed each month, how many searches are performed on Yahoo each year? Use the screen shots below to help determine the answer.



Lesson 13 Summary:

Lesson 13- Independent Practice

1. Write out a detailed proof of the fact that, given two numbers in scientific notation, $a \times 10^n$ and $b \times 10^n$, $a < b$, if and only if $a \times 10^n < b \times 1^n$.

a. Let A and B be two positive numbers, with no restrictions on their size. Is it true that $A \times 10^{-5} < B \times 10^5$?

b. Now, if $A \times 10^{-5}$ and $B \times 10^5$ are written in scientific notation, is it true that $A \times 10^{-5} < B \times 10^5$? Explain.

2. The mass of a neutron is approximately 1.674927×10^{-27} kg. Recall that the mass of a proton is 1.672622×10^{-27} kg. Explain which is heavier.

3. The average lifetime of the Z boson is approximately 3×10^{-25} seconds, and the average lifetime of a neutral rho meson is approximately 4.5×10^{-24} seconds.

a. Without using the theorem from today's lesson, explain why the neutral rho meson has a longer average lifetime.

b. Approximately how much longer is the lifetime of a neutral rho meson than a Z boson?