Algebra 1-2 Linear and Exponential Functions

Instructional Focus: Understand and use function notation

CCSS & Example	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Understand Relations of Functions (F.IF.1) ***Example***: Determine which of the following tables represent a function and explain why. Table A x | y 0 | 1 1 | 2 2 | 2 3 | 4 Table B r | T 0 | 0 1 | 2 1 | 3 4 | 5	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Determine and explain if a relation, graph <u>and</u> equation are functions.	Determine and explain if a relation, graph or equation are functions (<u>2 of the 3</u>).	Determine and explain if a relation, graph <u>or</u> equation are functions (<u>1 of the 3</u>).	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Use Function Notation (F.IF.2) ***Example:*** Let *f*(x) be the function that assigns to each minute after you placed the ham in the oven, its temperature in degrees Fahrenheit. Write a sentence for each of the following to explain what it means in everyday language. a. $f(0) = 65$ b. $f(5) < f(10)$ c. $f(40) = f(45)$		Use function notation and the values in the domain to calculate the values in the range <u>and interpret the value in context of a situation</u>.	Use function notation and the <u>values in the domain</u> to calculate the values in the range	Use function notation and <u>a given value</u> in the domain to calculate a values in the range	

F.IF.1 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of *f* is the graph of the equation y = f(x).

F.IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Algebra 1-2 Linear and Exponential Functions

Algebra 1-2 Linear and Exponential Functions

Instructional Focus: Construct linear and exponential functions

CCSS & Example	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Distinguish Between Linear and Exponential (F.LE.1*) Town A adds 10 people per year to its population, and town B grows by 10% each year. In 2006, each town has 145 residents. For each town, determine whether the population growth is linear or exponential. Explain.	Can extend thinking beyond the standard, including tasks that may involve one of the following: • Designing • Connecting • Synthesizing • Applying • Justifying • Critiquing • Analyzing • Creating • Proving	Explain whether a function is linear or exponential by describing its growth over intervals of equal width when analyzing a table, a graph, **and** function rule in context of a situation	**Explain whether a function is linear or exponential by describing its growth over intervals of equal width** when analyzing a table, a graph, **or** function rule in context of a situation	**Recognize** a linear or exponential function when analyzing a table, a graph, **or** function rule, in context of a situation	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Construct Linear and Exponential Functions (F.LE.2*) Albuquerque boasts one of the longest aerial trams in the world. The tram transports people up to Sandia Peak. The table shows the elevation of the tram at various times during a particular ride. Minutes into the ride: 2, 5, 9, 14 Elevation in feet: 7069, 7834, 8854, 10,129 Write an equation for a function that models the relationship between the elevation of the tram and the number of minutes into the ride.		Construct linear and exponential functions from a graph, verbal description and a table **in context of a situation**	Construct linear and exponential functions from a graph, verbal description **and** a table.	Construct linear and exponential functions from a graph, verbal description **or** a table.	
Rewrite and Interpret Functions (F.IF.8) The projected population of Delareyville is given by the function $p(t) = 1500(1.08)^{2t}$ where t is the number of years since 2010. You have been selected by the city council to help them plan for future growth. Explain what the function $p(t) = 1500(1.08)^{2t}$ means to the city council members.		Differentiate between exponential growth and exponential decay and identify the percent rate of change in exponential functions **in a context of the situation**	Differentiate between exponential growth and exponential decay and **identify the percent rate of change in exponential functions.**	Differentiate between exponential growth and exponential decay.	
Write Sequences (F.BF.1, F.BF.2, F.IF.3) A concert hall has 58 seats in Row 1, 62 seats in Row 2, 66 seats in Row 3, and so on. The concert hall has 34 rows of seats. a. Write a recursive formula to find the number of seats in each row. How many seats are in row 5? b. Write the explicit formula to determine which row has 94 seats?		Write arithmetic sequences recursively and translate to an explicit formula to model real world situations. Write geometric sequences recursively and translate to an explicit formula to model real world situations.	Write arithmetic sequences recursively and translate to an explicit formula. Write geometric sequences recursively and translate to an explicit formula.	Write arithmetic sequences in recursive or explicit form. Write geometric sequences in recursive or explicit form.	

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F.IF.3 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.

F.BF.1 Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process or steps for calculation from a context.

F.BF.2 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. *(Modeling Standard)

F.LE.1 Distinguish between situations that can be modeled with linear functions and with exponential functions. *(Modeling Standard)

a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.

b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.

c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

F.LE.2 Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table). *(Modeling Standard)

F.IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.

Algebra 1-2 Linear and Exponential Functions

Instructional Focus: Graphs of linear and exponential functions

<table>
<tr><th>CCSS & Example</th><th>4 – Mastery</th><th>3 – Proficient</th><th>2 - Basic</th><th>1 – Below Basic</th><th>0 – No Evidence</th></tr>
<tr>
<td>Interpret Key Features (F.IF.4*, F.IF.5)

Example: The local newspaper charges for advertisements in their community section. A customer has called to ask about the charges. The newspaper gives the first 50 words for free and then charges a fee per word. Use the table at the right to describe how the newspaper charges for the ads. Include all important information.
<table>
<tr><th># of words</th><th>Cost to place ad ($)</th></tr>
<tr><td>50</td><td>0</td></tr>
<tr><td>60</td><td>0.50</td></tr>
<tr><td>70</td><td>1</td></tr>
<tr><td>80</td><td>1.50</td></tr>
<tr><td>90</td><td>2</td></tr>
<tr><td>100</td><td>2.5</td></tr>
</table>
</td>
<td rowspan="3">Can extend thinking beyond the standard, including tasks that may involve one of the following:

• Designing
• Connecting
• Synthesizing
• Applying
• Justifying
• Critiquing
• Analyzing
• Creating
• Proving</td>
<td>Identify and interpret <u>all</u> key features in a table <u>and</u> graph in context of the situation.
• intercepts
• intervals where functions are increasing or decreasing
• intervals where the function is positive or negative
• end behavior
• domain

Translate a verbal description of a relationship to sketch a linear, <u>and</u> exponential graph.</td>
<td><u>Identify all and interpret at least 3</u> key features from a table <u>or</u> graph <u>in context of the situation.</u>
• intercepts
• intervals where functions are increasing or decreasing
• intervals where the function is positive or negative
• end behavior
• domain

Translate a verbal description of a graph's key features to <u>sketch</u> a linear <u>or</u> exponential graph.</td>
<td><u>Identify at least 3</u> key features from a table <u>or</u> graph
• intercepts
• intervals where functions are increasing or decreasing
• intervals where the function is positive or negative
• end behavior
• domain
Translate a verbal description of a graph's key features to <u>identify</u> a linear <u>or</u> exponential graph.</td>
<td rowspan="3">Little evidence of reasoning or application to solve the problem

Does not meet the criteria in a level 1</td>
</tr>
<tr>
<td>Graph Functions Symbolically F.IF.7</td>
<td>Graph linear and exponential functions expressed in symbolic form and show key features of the graph <u>in context of a situation.</u></td>
<td>Graph linear <u>and</u> exponential functions expressed in symbolic form and show key features of the graph</td>
<td>Graph linear <u>or</u> exponential functions expressed in symbolic form and show key features of the graph</td>
</tr>
<tr>
<td>Compare Key Features (F.IF.9*)

Example: David and Fred each throw a baseball into the air. The velocity of David's ball is given by $v(t) = 50 - 32t$ where v is in feet per second and t is in seconds. The velocity of Fred's ball is given in the table. What is the difference in the initial velocity between the two throws?
<table>
<tr><th>t</th><th>$v(t)$</th></tr>
<tr><td>0.2</td><td>38.6</td></tr>
<tr><td>0.4</td><td>32.2</td></tr>
<tr><td>0.6</td><td>25.8</td></tr>
<tr><td>0.8</td><td>19.4</td></tr>
<tr><td>1.0</td><td>13</td></tr>
</table>
</td>
<td>Compare key features of two functions represented
• algebraically
• graphically
• numerically in tables
• verbal descriptions
Key features include:
• intercepts
• rate of change
• increasing or decreasing
• <u>positive or negative</u>
• <u>end behavior</u></td>
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Algebra 1-2 Linear and Exponential Functions

Calculate and Interpret Rate of Change (F.IF.6*, F.LE.3) Draining Water from a Hot Tub Time (s) / Volume (L) 0 / 1600 10 / 1344 20 / 1111 30 / 900 40 / 711 50 / 544 60 / 400 70 / 278 80 / 178 90 / 100 100 / 44 110 / 11 120 / 0 The plug is pulled in a small hot tub. The table gives the volume of water in the tub from the moment the plug is pulled, until it is empty. What is the average rate of change between: 60 seconds and 100 seconds? 0 seconds and 120 seconds?		Calculate the average rate of change over a given interval and explain the meaning in context for linear and exponential functions presented in symbolic, table and graph form Describe that an increasing exponential function will eventually exceed a linear function	Calculate the average rate of change over a given interval and explain the meaning in context for linear and exponential functions presented in symbolic, table or graph form	Calculate the average rate of change over a given interval for linear and exponential functions presented in symbolic, table or graph form
Interpret Parameters of a Function (F.LE.5*) Lauren keeps records of the distances she travels in a taxi and what it costs: Distance d in miles / Fare f in dollars 3 / 8.25 5 / 12.75 11 / 26.25 a. If you graph the ordered pairs (d,f) from the table, they lie on a line. How can this be determined without graphing them? b. Show that the linear function in part a. has equation F =2.25d + 1.5. c. What do the 2.25 and the 1.5 in the equation represent in terms of taxi rides?		Explain the meaning of ● exponential function's base, end behavior, and rate of growth in context ● linear function's slope and intercepts in context. ● coefficients, factors, exponents, and intercepts in a linear or exponential function.	Explain the meaning of 2 ● exponential function's base, end behavior, and rate of growth ● linear function's slope and intercepts ● coefficients, factors, exponents, and intercepts in a linear or exponential function.	Explain the meaning of 1 ● exponential function's base, end behavior, and rate of growth ● linear function's slope and intercepts ● coefficients, factors, exponents, and intercepts in a linear or exponential function.
Find Solutions Graphically (A.REI.11) The functions $f(m) = 18 + 0.4m$ and $g(m) = 11.2 + 0.54m$ give the lengths of two different springs in centimeters, as mass is added in grams, m, to each separately. a. Graph each equation on the same set of axes. b. What mass makes the springs the same length? c. What is the length at that mass? d. Write a sentence comparing the two springs.		For linear and/or exponential functions, find intersection points using technology, graphs, and tables and explain in the context of a situation	For linear and/or exponential functions, find intersection points using technology, graphs, and tables	For linear and/or exponential functions, find intersection points using technology, graphs or tables

Algebra 1-2 Linear and Exponential Functions

F.IF.4 For a linear, exponential, or quadratic function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; and end behavior. *

F.IF.5 Relate the domain of a linear, exponential, or quadratic function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. *

F.IF.6 Calculate and interpret the average rate of change of a linear, exponential, ~~or quadratic~~ function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph of a function over a specified interval. *

F.LE.3 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, ~~quadratically, or (more generally) as a polynomial function.~~ *(Modeling Standard)

F.IF.9 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

F.LE.5 Interpret the parameters in a linear or exponential function in terms of a context. *(Modeling Standard)

A.REI.11 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find solutions to f(x) = g(x) approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, quadratic, or exponential functions. *(Modeling Standard)

F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

a. Graph linear ~~and quadratic~~ functions and show intercepts, maxima, and minima.

e. Graph exponential ~~and logarithmic~~ functions, showing intercepts and end behavior~~, and trigonometric functions, showing period, midline, and amplitude~~