Instructional Focus: Represent and calculate with vectors

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Represent vectors (N.VM.1)	Meets **all** of the criteria in a Level 3 **Justify solutions and critique the reasoning of others**	Use appropriate symbols for vectors and their magnitude, represent vector quantities by directed line segments, **and find the magnitude and direction of vector quantities.**	Use appropriate symbols for vectors and their magnitude and **represent vector quantities by directed line segments**.	Use appropriate **symbols for vectors and their magnitude**.	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Solve problems with vectors (N.VM.3)		Solve problems involving velocity and other quantities by converting given direction and magnitude quantities into component vectors, calculate the resultant vector, **and find the resultant direction and magnitude or the angle between vectors**	Solve problems involving velocity and other quantities by converting given direction and magnitude quantities into component vectors**, and calculate the resultant vector**	Solve problems involving velocity and other quantities by **converting given direction and magnitude quantities into component vectors**	
Operations with vectors (N.VM.2, N.VM.4, N.VM.5)		Find the components of a vector by subtracting coordinates Add, subtract vectors graphically and component-wise, **and determine the magnitude and direction** Multiply a vector by a scalar and **determine the magnitude and direction**	Find the components of a vector by subtracting coordinates Add, subtract vectors graphically **and** component-wise Multiply a vector by a scalar	Find the components of a vector by subtracting coordinates Add, subtract vectors graphically **or** component-wise Multiply a vector by a scalar	

Pre-Calculus – Coordinate Systems

N.VM.1 (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).

N.VM.3 (+) Solve problems involving velocity and other quantities that can be represented by vectors.

N.VM.2 (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.

N.VM.4 (+) Add and subtract vectors.

a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.

b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.

c. Understand vector subtraction $\boldsymbol{v} - \boldsymbol{w}$ as $\boldsymbol{v} + (-\boldsymbol{w})$, where $-\boldsymbol{w}$ is the additive inverse of $\boldsymbol{w}$, with the same magnitude as $\boldsymbol{w}$ and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.

N.VM.5 (+) Multiply a vector by a scalar.

a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$.

b. Compute the magnitude of a scalar multiple $c\boldsymbol{v}$ using $||c\boldsymbol{v}|| = |c|v$. Compute the direction of $c\boldsymbol{v}$ knowing that when $|c|v \neq 0$, the direction of $c\boldsymbol{v}$ is either along $\boldsymbol{v}$ (for $c > 0$) or against $\boldsymbol{v}$ (for $c < 0$).

Instructional Focus: Represent and calculate complex numbers

	4 – Mastery	3 – Proficient	2 - Basic	1 – Below Basic	0 – No Evidence
Represent on the complex plane (N.CN.4)	Meets **all** of the criteria in a Level 3 **Justify solutions and critique the reasoning of others**	Represent complex numbers on the complex plane in rectangular and polar form, **and explain** why the rectangular and polar forms of a given complex number represent the same number	Represent complex numbers on the complex plane in rectangular **and polar form**	Represent complex numbers on the complex plane in **rectangular form**	Little evidence of reasoning or application to solve the problem Does not meet the criteria in a level 1
Operations of Vectors in Polar Form (N.CN.3, N.CN.5)		Represent and compute addition and subtraction of complex numbers geometrically on the complex plane Represent and compute multiplication and division, in polar form, of complex numbers geometrically on the complex plane **Represent and compute the power and roots of complex numbers, in polar form.**	Represent and compute addition and subtraction of complex numbers geometrically on the complex plane **Represent and compute multiplication and division, in polar form, of complex numbers geometrically on the complex plane**	Represent and **compute addition and subtraction** of complex numbers geometrically on the complex plane	
Calculate distance and midpoint (N.CN.6)		Calculate the **distance between numbers in the complex plane as the modulus of the difference**, and calculate the midpoint of a segment in the complex plane as the average of the numbers at its endpoints	Calculate the **difference between numbers in the complex plane**, and calculate the midpoint of a segment in the complex plane as the average of the numbers at its endpoints	Calculate the **midpoint of a segment in the complex plane** as the average of the numbers at its endpoints	

Pre-Calculus – Coordinate Systems

N.CN.4 (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

N.CN.3 (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

N.CN.5 (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\, i)^3 = 8$ *because* $(-1 + \sqrt{3}\, i)$ *has modulus* 2 *and argument* 120°.

N.CN.6 (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.